Ekonomia nr 33/2013

Comparison of Alternative Approaches to VaR Evaluation

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Abstract

The main goal of this article is to present alternative methods of market risk measurement in Polish banking sector with popular Value at Risk (VaR) approach. Four main methods: analytical, historical, simulation and hybrid (Filtered Historical Simulation, FHS) of VaR are presented and then three of them are applied to evaluate interest risk stemming from government bonds' portfolio held by Polish banks. Adequacy of VaR measures counted with particular methods is compared with the help of formalized criteria and best fitted methodology is recommended.

Keywords: market risk, VaR, analytical, historical, simulation and hybrid VaR.

JEL Code: G11, G12.

Introduction

Market risk is one of the main types of risks to which financial institutions are exposed. In the case of investment banks, which hold large trading books, this class of risk can be perceived as a substantial one, in parallel with counterparty (credit) and liquidity risk. In a Polish banking sector, dominated with universal banks, market risk is not perceived as a main threat to stability of the sector. Polish banks' trading book is packed mainly with local government and central bank debt instruments, what makes it easier to hedge and manage. However it should

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be considered that these exposures still constitute a big fraction of local banks assets and in a case of distress on financial markets can contribute to instability of these institutions.

The main goal of this article is to present methods of market risk measurement in Polish banking sector with popular Value at Risk (VaR) approach. Four main methods: analytical, historical, simulation and hybrid (Filtered Historical Simulation) of VaR simulation are presented and then three of them are applied to simulate interest risk stemming from debt portfolio held by Polish banks. The quality of VaR measures counted with particular methods is compared using formalized criteria and finally best fitted methodology is recommended.

The rest of the article is organized as follows. In the first part literature of the market risk estimation is discussed and market risk definition is presented. Next, general idea of Value at Risk method is described and four main approaches to its estimation are discussed in detail. Empirical part of the paper is devoted to description of portfolio's description, which mimicked general structure of Polish banks trading book portfolio. Finally simulation exercise is briefly described and achieved market risk measures and their characteristic are discussed.

1. Literature overview

Definition of market risk and general description of its measures evaluation can be found in the works of Jajuga (2000, 2007a, 2007b) and in the book of Hull (2007). Value at Risk method (and its application) is broadly discussed in the articles of Duffy and Pan (1997), Holton (2003), Dowd (2005) and Danielsson (2011). Alternative methods of VaR estimation are also presented in the works of Jorion (2007), Hull and White (1998) and Prisker (2006) (Filtered historical) simulation approach to VaR modelling is explained in detail the works of Glasserman (2004) and Mentel (2012).

The VaR methodology gained its popularity thanks to series of Basel Committee of Banking Supervision (BCBS) recommendations introduced to the global banking sector almost twenty years ago. The information about history of market risk regulations' implementations, methodology of VaR application in the banking sector market risk assessment and general market risk management is enclosed in series of BCBS reports and manuals (BCBS, 1996, 2005, 2008).

2. Definition of market risk

Market risk is defined in the literature as a risk generated with financial instruments' price changes observed on the primary or derivative markets (Jajuga, 2007). These instruments are held as assets by financial institutions, hence situation on the financial markets influences condition of these entities with mark-to-market (MtM) mechanism. Taking into account types of instruments which are traded on financial markets four main types of market risk can be distinguished:

1. Interest rate (IR) risk
2. Foreign exchange (FX) risk
3. Stock price risk
4. Commodity price risk

Polish banks' traditional business model makes them prone mainly to interest rate risk According to Jajuga (2007) this type of risk can be determined as a risk stemming from changes of assets or liabilities prices due to interest rates' fluctuations. In the practical part of the survey debt instruments' portfolio mimicking Polish banks expositions will be built to compare different Value at Risk measures.

3. Introduction to VaR

Value at Risk (VaR) of a financial instruments portfolio is defined as maximum value of the loss, which potentially can be taken by its owner in a certain time horizon with certain confidence level. From statistical point of view VaR can be defined as a quantile risk measure. For a given continuous distribution of stochastic variable R representing return from mentioned portfolio and quantile q: $q \in [0, 1]$ VaR can be computed as a certain number such as:

$$P(R < VaR_q) = q. \tag{1}$$

Hence if for quantile q = 1% VaR of a portfolio of assets' returns in the perspective of 10 days is reported to be –3 PLN million, it can be assumed that maximal loss from this portfolio in a given horizon with probability 0.99 should not exceed mentioned VaR value. If the distribution function of the portfolio returns is known (F(·)), VaR of the portfolio for a quantile q at a time t is given as:

$$VaR_{t,q} = F^{-1}(q). \tag{2}$$

To compute VaR measure of market risk in practice very often strict assumptions about variable R and structure of analysed portfolio are taken:

1. Variable R should be identically and independently distributed across the time (*i.i.d.*);
2. Variable R should have Gaussian distribution;
3. Amount of a portfolio's instruments should be static (buy-and-hold strategy and static VaR) or weights of the portfolio's components should be adjusted for changes of the instruments' value changes (dynamic VaR);
4. For changing scale of VaR from one perspective to another one, rule of square root should be applied.

If first two assumptions are fulfilled ($R^{i.i.d.} \sim N(\mu,\sigma)$) VaR of the portfolio can be computed at a time t according to the formula:

(3) $$VaR_{t,q} = \sigma\Phi^{-1}(1-q) + \mu,$$

where $\Phi(\cdot)$ is Gaussian distribution.

The fourth assumption is derived from first three ones. It defines the way in which the time perspective of VaR is switched. If the portfolios' returns are Gaussian *i.i.d.* and portfolio components' weights are constant across the time (dynamic portfolio) the 1-period VaR can be scaled to h-period VaR with the formula:

(4) $$VaR^{h}_{t,q} = \sqrt{h}\sigma\Phi^{-1}(1-q) + h\mu.$$

Breaking mentioned above assumptions makes usually VaR risk measure unreliable. Therefore checking assumptions fulfilment of the reported VaR (especially first assumption) is one of the most important tasks of the analyst applying this measure in practice.

4. Different approaches to VaR evaluation

Methods of VaR computation can be classified into four main groups of methods:

1. Analytical;
2. Historical;
3. Simulation;
4. Hybrid.

In the following sections methods of practical application will be discussed.

4.1. Analytical VaR

Portfolio's VaR can be computed with help of analytical approach if its return is linear function of the market risk factors, which affect this portfolio. Moreover if the particular portfolio's components' returns are Gaussian and total portfolio returns are Gaussian, its VaR depends only on variance/covariance matrix of these factors. Beside that in analytical VaR computation portfolio's total risk measure is decomposed into general market risk VaR (called also systemactic VaR) and VaR stemming from risk factors connected with particular components of the analysed portfolio (idiosyncratic VaR). Systematic VaR computation requires building analytical structure, which will bind portfolio's total return with certain risk factors affecting portfolio's value. This structure is called risk factor projection.

Let's assume that total portfolio return at a time t is function of returns associated with risk factors (*RF*) classified into *C* classes ($c = 1, 2, \ldots, C$) affecting *I* component instruments ($i = 1, 2, \ldots, I$). The projection of the portfolio on these risk factors can be written as:

$$R_t = F(RF_t^{c,i}). \tag{5}$$

If considered portfolio's return can be perceived as linear combination of the components' returns, the total portfolio may be computed as weighted average of all risk factors belonging to *C* classes:

$$R_t = \sum_{c=1}^{C} \sum_{i=1}^{I} a_t^{c,i} R_t^{c,i}, \tag{6}$$

where $a_t^{c,i}$ represents total portfolio's return sensitivity to risk factor *c* driving return of portfolio *i* ($R_t^{c,i}$). Computation of h-days perspective portfolio's VaR requires knowledge of expected values and (co)variance (μ^h, Σ^h) of portfolio's components' returns implied by contribution of particular risk factors.

In such a situation expected value and (co)variance of portfolio returns at a moment t are given respectively as:

$$E(R_t^{c,i}) = \Psi_t^h \mu^h; \quad V(R_t^{c,i}) = \Psi'^h_t \Sigma^h \Psi_t^h, \tag{7}$$

where Ψ_t^h is vector grouping sensitivities $a_t^{c,i}$.When it is additionally assumed that returns distributions associated with risk factors belonging to particular classes have normal distribution, *q*-quantile systematic VaR in a *h*-periods perspective is given as:

$$systematic\ \ VaR_{t,q}^h = \Phi^{-1}(1-q)\sqrt{\Psi'^h_t \Sigma^h \Psi_t^h} + \Psi_t^h \mu^h. \tag{8}$$

For scaling purposes the dependence between 1-period and *h*-period systematic VaR is determined with expression:

$$systematic\ \ VaR_{t,q}^h = \sqrt{h}\, VaR_{t,q}^1. \tag{9}$$

If we consider two classes of systematic risk (e.g. interest rate and foreign exchange risk) vector of risk factor sensitivities can be decomposed into two subgroups. Each subgroup represents risk factors belonging to one of these groups:

$$\Psi_t^h = \left(\Psi_t^{RF\,1,h}, \Psi_t^{RF\,2,h}\right), \tag{10}$$

where $\Psi_t^{RF1,h}$,$\Psi_t^{RF2,h}$ represent sensitivities' vectors associated with first and second group respectively.

In practice it is usually assumed that sensitivities coefficients for individual groups are given as:

(11) $$\Psi_t^{RF1,h} = \left(PV01_t^{RF1,1}, PV01_t^{RF1,2}, \ldots, PV01_t^{RF1,\mathrm{I}}\right),$$

(12) $$\Psi_t^{RF2,h} = \left(PV01_t^{RF2,1}, PV01_t^{RF2,2}, \ldots, PV01_t^{RF2,\mathrm{I}}\right) PV_t^{-1},$$

where $PV01_t^{RFX,i}$ responds to present values of the basis points connected with instrument i and PV_t is value of the portfolio at a time t. Grouping risk factors according to particular risk classes should be also connected with partition of the covariance matrix:

(13) $$\Sigma^h = \begin{bmatrix} \Sigma^{F1,h} & \Sigma^{F1,F2\,h} \\ \Sigma^{F1,F2,h} & \Sigma^{F2,h} \end{bmatrix},$$

where $\Sigma^{F1,F2,h}$ is covariance matrix of risk factors belonging to different classes. Using this matrix total exposition to all risk types as well as exposition stemming from individual risk factors can be computed (in the sample below VaR connected with risks factor $F1$ is presented):

(14) $$F1\,VaR_{t,q}^h = \Phi^{-1}(1-q)\sqrt{\Psi'^{F1,h}_t \Sigma^{F1,h} \Psi_t^{F1,h}} + \Psi_t^{F1,h} \mu^{F1,h}.$$

It should be added there that sum of VaR stemming from particular risk classes is usually not equal to the total VaR of the portfolio. The equality is hold only if risk factors are perfectly correlated with each other.

Contributions of particular risk factors to total portfolio's VaR are computed with incremental VaR. Incremental VaR is approximated with setting sensitivities to other risk factors (other than reference factor) to zero. Transformed sensitivity vector $\Psi_t^{0,h}$:

(15) $$incremental\ \ VaR = \Psi'^{0,h}_t g(\Psi_t^{0,h}).$$

where $g(\cdot)$ is vector of total VaR gradients (elements of this vector are derivatives of the total VaR respective to components of Ψ_t^h):

(16) $$g(\Psi_t^{0,h}) = \Phi^{-1}(1-q)\Sigma^h \Psi_t^h \sqrt{\Psi'^h_t \Sigma^h \Psi_t^h}.$$

In practical computations risk factors are extracted with principal components analysis (PCA) or factor analysis (FA) (where dependence between portfolio's return and risk factors is static) or Dynamic Factor Models (DFM) (if this relation is dynamic).

4.2. Historical VaR

Historical VaR (also known as Historical Simulation VaR, HS VaR) is response to the limits of analytical version of this measure. Strict assumptions which are usually taken while computing analytical VaR (e.g. *i.i.d.* Gaussian distributions of portfolio returns) make this method often unreliable when applied to empirical portfolios.

Historical simulation eliminates part of these strict, often counterfactual assumptions. First of all to compute VaR with historical simulation it is not necessary to determine *ex ante* distribution of analysed portfolio's total return (and returns of its components). In contrary empirical distributions (based on historical data) are applied in the computations. Moreover historical simulation VaR frees analysts from assumption about independence of portfolio's total return across the time. Hence econometrical methods, which allow modelling volatility clusters and time series autocorrelation (e.g. class of GARCH models), can be applied here. However it should be remembered that HS VaR requires keeping assumption of total return's identical distribution.

Unfortunately there are some disadvantages of historical VaR. Achieving high quality of empirical distributions of portfolio's total return requires long time series of portfolio components returns (prices or yields). It could be very restrictive requirement, especially when we consider emerging markets, where quotations of some instruments' prices have very short history. As a minimal standard for HS simulation one year of daily observations (approximately 250 observations) should be used. It is also hard to support the assumption, that (especially in a long-term perspective) distributions of portfolio returns are identical.

Computation of simplest version of historical VaR requires additional assumption that portfolio is linear combination of its components and that weights of the portfolio's components are constant across the time. Considering this assumption we can express portfolio's total return at time t as a weighted (with weights w_i, $i = 1, 2, \ldots, N$) returns ($R_{i,t}$) of its components:

$$R_t = \sum_{i=1}^{N} w_i R_{i,t} . \tag{17}$$

Having M historical realizations of stochastic variables drawn from unknown distributions and applying assumption about their stability across the time, that total return density function "forecasted" at a time t for one period ahead can be approximated with weighted frequency function computed as sequences of M observations of each N individual portfolio components quotations:

$$\{R_{t+1|t}\} \approx \left\{\sum_{i=1}^{N} w_i R_{i,t-1-p}\right\}_{p=1}^{M} . \tag{18}$$

q-quantile historical one-period ahead "forecast" VaR is computed with sorted ascending portfolio's total returns drawn from empirical approximation of empirical distribution and then choosing such $VaR_{t+1,q}$ level that 100%*q observations were smaller than this number:

$$VaR_{t+1,q} = Q_t^q\left(\{R_{t+1|t}\}\right) . \tag{19}$$

Applying 1-period ahead VaR in practice requires establishing rule of scaling this measure for h-periods ahead. Taking into account lack of Gaussian distribution assumption "square root" rule used for analytical VaR is inadequate here. Scaling quantile measures derived from non-normal distributions requires keeping *i.i.d.* assumption of portfolio's total returns, what approximately can be interpreted as assumption about α-stability of this distribution. For an α-stabile distribution coefficient of quantile measure scaling is equal to $l^{1/\alpha}$:

$$Q_{t,p}^{l} = l^{1/\alpha} Q_{t,p}^{l} . \tag{20}$$

Basic HS VaR models are biased with important limitation implied by great number of time series observations, which are needed to achieve reliable theoretical portfolio's total return approximation. On the other side, applying long time series to the distribution estimation rises other problem connected with exceeded influence of "old" historical observations for present situation. As a solution of this problem Weighted Historical Simulation VaR (WHS VaR) is usually used. Main idea of this method is to assign, with the exponential decay schema, greatest weights for recent observations. According to this schema for each of the M observations with index p used for computation of empirical return recursive weights are assigned. These weights (Exponential Decay Weight, EDW) depend on "time distance" of the observation indexed p from a most recent observation (indexed t):

$$edw_p = edw_{p-1}\left(\frac{1-edp}{1-edp^M}\right), \tag{21}$$

where edp stands for exponential decay parameter. Disadvantage of this approach is arbitral selection of mentioned parameter value (usually the value near one is set). Due to lack of formalized selection procedure this parameter is very often selected with "rule of thumb".

4.3. Simulation (Monte Carlo VaR)

Simulation VaR (known also as Monte Carlo VaR) has some unquestionable advantage, namely possibility of modelling non-standard portfolio's return distribution, which can be useful if analysts expect extreme changes in a market environment in a nearest future. Simulation VaR allows also to model market risk distribution stemming from nonlinear portfolios (the portfolio consisting of instruments with nonlinear payments, e.g. options). Moreover Monte Carlo method used usually for this type VaR computation is intuitive and easy to implement in practice.

Of course there are also some disadvantages of introduced method. First of all simulations used to compute this kind of VaR can be biased with high errors. Majority of simulation error reduction methods proposes increase of simulation number as a panacea for this problem, what leads to another disadvantage of simulation VaR, namely computation complexity. MC VaR requires also taking initial assumption about functional form of portfolio's distribution, what makes this method (like analytical VaR) prone to model specification risk. Moreover simulation VaR is perceived sometimes as strictly hypothetical approach. Selecting initial portfolio's return distribution analysts take very often too liberal, or from the other side, too restrictive assumptions about future shape of this distribution. To conclude this part it however should be mentioned that simulation VaR is very flexible analysis tool and one of the few available when it comes to modelling expected extreme changes of market conditions.

Next passage will be devoted to description of simulation VaR computation (h-periods ahead) with application of Monte Carlo method. First stage of this procedure is to determine volatility of modelled portfolio's returns. It is usually done with simple GARCH(1,1) model specification:

$$R_t = \sigma_t x_t, \tag{22}$$

where x_t is stochastic variable drawn from certain (defined by analyst) distribution. Beside that functional form of the portfolio's return's volatility evolution need to be determined. Continuing simplifying approach inter-period volatility evolution can be described with the formula:

$$\sigma_{t+1}^2 = a + bR_t^2 + c\sigma_t^2. \tag{23}$$

In the next step NMC simulated values of determined previously portfolio's return distribution need to be determined $x_{t+1,i}$, $i = 1, 2, ..., N^{MC}$. In practical applications pseudorandom number generators are used. To diminish influence of approximation of empirical distribution with theoretical one number of simulations is established to be more or equal to 1000. Taking into account actual computing power of desktop computers number of simulations used for market risk assessment is set to be 10.000 or even more.

In the next step, values of $x_{t+1,i}$ are applied in equations (22)/(23) and R_{t+2} with σ^2_{t+2} are computed. Steps number 2) and 3) are iteratively repeated h-times to compute sequences of one-step-ahead portfolio's returns and variances adequate for considered time intervals, starting from "one-period-ahead" return at time t:

$$(24) \qquad \left(\hat{R}_{t+j,i}\right)_{i=1}^{N^{MC}}, \quad j = 1, 2, \ldots, h.$$

Using this sequence for each $i = 1, 2, \ldots, N^{MC}$ cumulative portfolio's return for time span from $t + 1$ to $t + h$ is computed:

$$(25) \qquad \hat{R}_{t+1:\,t+h,i} = \sum_{j=1}^{h} \hat{R}_{t+j,i}\,.$$

These N^{MC} cumulated portfolio's returns are used to build their distribution:

$$(26) \qquad \left\{\hat{R}_{t+1:\,t+h,i}\right\}_{i=1}^{N^{MC}},$$

which finally is used to compute q-quantile h-periods ahead simulated VaR at time t:

$$(27) \qquad MCVaR_{t+h,q} = Q_t^h\left(\left\{\hat{R}_{t+1:\,t+h,i}\right\}_{i=1}^{N^{MC}}\right).$$

4.4. Hybrid method

Historical and simulation VaR evaluation approach can be combined into Filtered Historical Simulation VaR (FHS VaR). This method's main idea was to keep key advantages of its origins. In practice it uses formalized models of portfolio's return variance and empirical approach to portfolio's return distribution modelling. Analogously to Monte Carlo VaR portfolio's return and variance are described with GARCH(1,1) like equations. These equations are applied to estimate historical portfolio's return variance (σ_{t-p}), which are used next (as standard deviations) to compute residuals from the equation (22) (interpreted as standardized portfolio's returns):

$$(28) \qquad x_{t-p} = \frac{R_{t-p}}{\hat{\sigma}_{t-p}}, \quad p = 1, 2, \ldots, M.$$

In the next part of FHS VaR, similarly to MC VaR approach, in h steps one-step-ahead “forecast” of portfolio’s returns variance and portfolio’s returns are counted alternately. In contrary to simulation VaR scenarios are not generated with a priori selected distribution, but in each step they are drawn N^{FHS} times from sequences of historical residuals computed from equation (28) $(x_{t-p})^{M}{}_{p=1}$.

As a result of above bootstrapping exercise N^{FHS} h-elements sequences of one step ahead “forecasts” of portfolio’s returns:

$$\left(\breve{R}_{t-j,i}\right)_{i=1}^{N^{FHS}}, \quad j = 1, 2, \ldots, h. \tag{29}$$

Next steps mimic the computation sequence described for Monte Carlo VaR. Sequences (29) are used to compute cumulative portfolio’s returns for a time span between points $t+1$ and $t+h$:

$$\breve{R}_{t+1;\, t+h;i} = \sum_{j=1}^{h} \breve{R}_{t+j,i}, \tag{30}$$

which are used to build distribution:

$$\left\{\breve{R}_{t+1:\, t+h,i}\right\}_{i=1}^{N^{MC}}, \tag{31}$$

applied finally to count q-quantile h-periods ahead filtered historical simulation VaR at time t:

$$FHSVaR_{t+h,q} = Q_t^h\left(\left\{\breve{R}_{t+1:\, t+h,i}\right\}_{i=1}^{N^{MC}}\right). \tag{32}$$

5. Data

In the practical part of the survey comparison of VaR calculation methods was performed. In the first step portfolio of financial instruments was constructed. Trying to apply results of this survey for further application to Polish banking sector, interest rate risk stress testing portfolio of debt instruments was built. The structure of this portfolio mimicked structure of market assets held in the last quarter of 2010 by forty biggest Polish banks. The information about the structure was taken from microprudential database of Narodowy Bank Polski.

Taking into account fraction of debt issued by residents (99% of total Polish banks’ bonds portfolio) only these papers were considered in the analysis. Next banks’ expositions to interest risk from this portfolio were mapped into five time structure buckets (0-6M), (6M-3Y), (3Y-7Y), (7Y and more). Information about banks' hedging positions (interest rate swaps, future tare agreements) from off-balance sheet statements were taken into consideration as well. In practical computations daily adjustments of portfolio structure were used to keep constant portfolio weights. Ideal partition of instruments and lack of the transaction costs was assumed as well.

Table 1. Structure of 40 biggest Polish banks debt instruments portfolio in the last quarter of 2010.

Issuer's type	Resident	Non-resident
Fraction of residents/nonresidents' papers	99%	1%
Fraction of issuer's type in a whole portfolio		
Central banks (money bills)	34%	0%
Central government institutions	56%	25%
– of which treasury bills	6%	0%
– of which government bonds	50%	25%
Local government institutions	4%	0%
Other financial institutions	3%	70%
Other non-financial institutions	3%	5%

Source: NBP data.

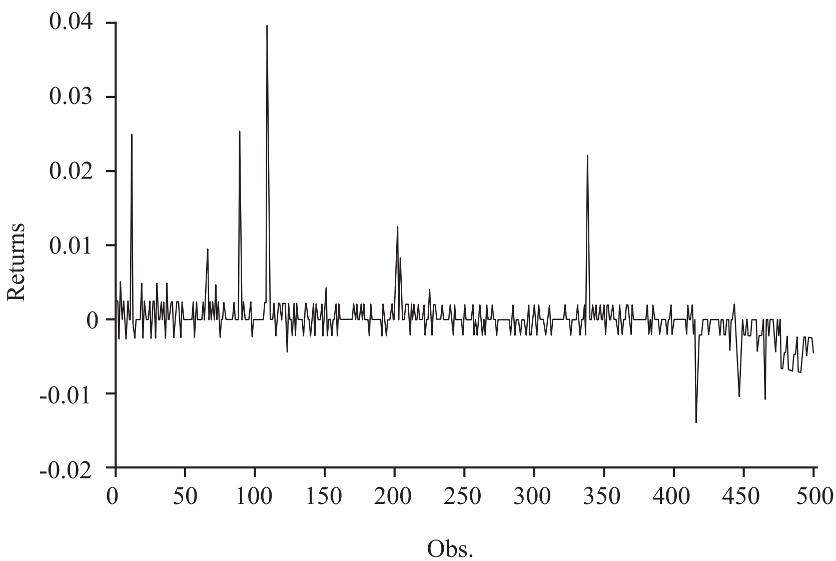

Figure 1. Log. returns from WIBOR3M.
Source: Bloomberg.

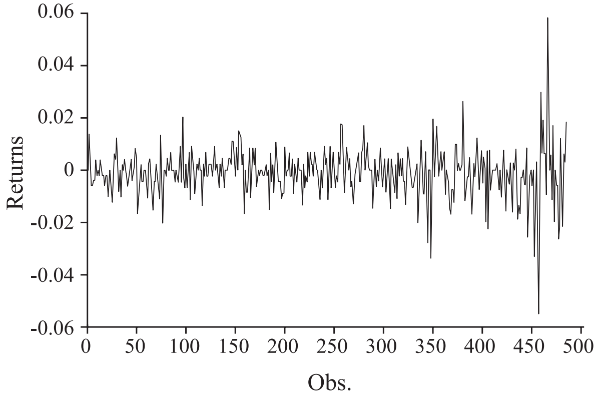

Figure 2. Log. returns from 2Y government bonds.
Source: Bloomberg.

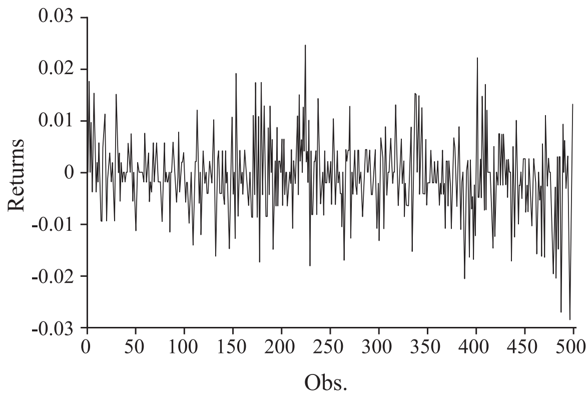

Figure 3. Log. returns from 5Y government bonds.
Source: Bloomberg.

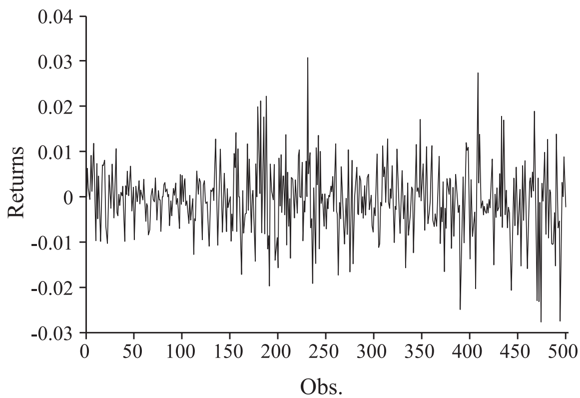

Figure 4. Log. returns from 10Y government bonds.
Source: Bloomberg.

Taking into account portfolio's time structure mapping four main factors of interest rate risk were identified: short term interest rate (WIBOR 3M), two mid-term interest rates (2Y and 5Y government bonds' yields) and long term interest rate (10Y government bonds' yield). Next four figures shows log returns from particular risk factors.

In the next table basic descriptive statistics of four selected interest rate risk factors were presented.

Table 2. Descriptive statistics of selected risk factors

	WIBOR3M	2Y bonds	5Y bonds	10Y bonds
Mean	0.0001	-0.0009	-0.0010	-0.0009
Median	0.0000	0.0000	0.0000	0.0000
Max	0.0395	0.0584	0.0245	0.0307
Min	-0.0138	-0.0551	-0.0284	0.0309
Standard dev.	0.0034	0.0094	0.0071	0.0077
Skewness	4.8375	0.1591	-0.1384	-0.1051
Kurtosis	52.2988	11.2815	4.4236	4.7809
Jarque-Bera statistics	52583	1247.8	43.7	67.8
J-B probability	0.000	0.000	0.000	0.000
Number of observations	500	500	500	500

Source: own computations.

Using the result we can find the most volatile instrument, 2Y governments bonds. For all instruments excessed kurtosis was noticed ($\kappa > 3$) what could be explained with extreme changes of selected instruments' log returns. No instrument passed Jarque-Bera test, what meant that for all of them Gaussian distribution hypothesis had to be rejected.

6. Simulations

In the practical part of the survey three methods of VaR computations were compared: Analytical VaR (AVaR), Historical VaR (HVaR), Filtered Historical Simulation VaR (FHSVaR). For all of these methods computation window length had to be chosen. Considering length of the sample (500 observations) window size was set to 100. Hence first VaR value was computed for observation number 101 (fixed point approach) and over for $T = 400$ following observations. Moreover confidence level of all VaRs was set to 0.99, what reflects tightened conditions of market risk evaluation after 2008.

The comparison of methods was performed with help of two criteria, both of them exploited quantile characteristics of VaR measure. For each method empirical number of VaR exceedances percentage was compared with quantile corresponding to assumed VaR confidence level (0.99 corresponding 1st percentile).

This criterion measured bias of estimated VaR measures with theoretical 1% threshold. As a second criterion Kolmogorov-Smirnov test was used to check presence of autocorrelation of VaR exceedances (for 50 lags). With this criterion presence of systematic VaR estimation error was checked.

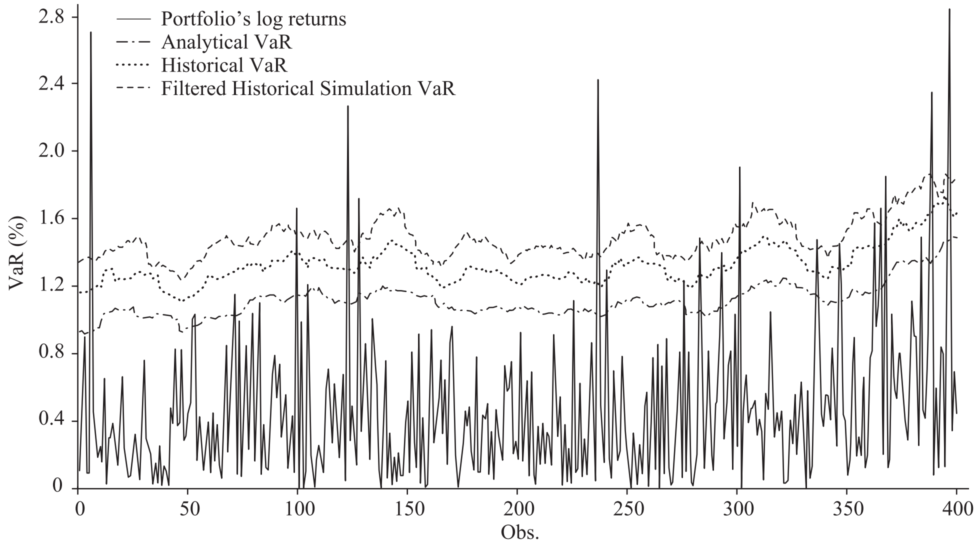

Figure 5. Absolute Log returns of analysed portfolios and VaR computed with three alternative methods, computation window size equals to 100.
Source: own computations.

7. Results

In the Figure 5 absolute log returns from the constructed portfolio's returns and VaRs computed with three alternative methods were presented.
Based on the figure above it can be noticed that the highest values of VaR is generated by Filtered Historical Simulation Method (FHSVaR), lowest from the analytical VaR (AVaR). Looking at empirical percentage of exceedances (Table 3) of particular VaR methods it seems that highest observations of 1st quantile VaR computed with FHS VaR are the most adequate ones. Combination of historical approach and Monte Carlo simulation implies that empirical number of excessed VaR differs from the theoretical one only by 0.25%. It is highly probable that two other methods (Analytical VaR and Historical VaR /HVaR/) underestimate portfolio's market (interest rate) risk measure, in the case of AVaR the number of empirical exceedances is almost two times higher than desired one.

Table 3. Number of exceedandes for alternate VaRs

VaR type	AVaR	HVaR	FHSVaR
Empirical number of VaR exceedances	1.25%	1.5%	2.25%

Source: own computations.

When it comes to VaR response to changes of portfolio's returns it can be noticed that methods based on historical approach (HVaR and FHSVaR) respond a bit quicker to fluctuation of returns that the analytical VaR. However the difference between these methods is not significant.

In the last exercise Kolmogorov-Smirnov test was performed to check presence of autocorrelation in empirical VaR exceedances. As it can be seen in the Table 4 for all used methods null hypothesis stating autocorrelation presence can be rejected as not statistically significant. It should be however remembered that these statistics are computed on very small numbers of observation (less than 10), hence reported results can be misleading to the some point.

Table 4. Kolmogorov-Smirnov test results for alternate VaRs (statistics and p-values)

VaR type	AVaR	HVaR	FHSVaR
Statistics	0.47	0.019	0.072
p-value	0.0000	0.0002	0.0000

Source: own computations.

Conclusions

In this paper author tried to present different approaches to Value at Risk (VaR) market risk measure estimation based on practical survey performed with help of portfolio of debt instruments with similar structure to portfolio possessed by Polish banks. For analytical, historical, simulation and hybrid methods, theory of VaR estimation was depictured and then three methods were applied in practical exercise to estimate interest rate risk stemming from trading books of domestic banks.

Gained results proved superiority of Filtered Historical Simulation (FHS) VaR method, which is perceived as a hybrid approach. This method allows combining advantages of two origin methods: historical and simulation method. The surveys have shown that VaR, computed as interest risk measure is the most adequate in the case of FHS model. This method showed lowest VaR exceedance numbers and responded very quickly to changes of portfolio's returns volatility. Simultaneously the thesis of limited influence of market risk on Polish banks was proved.

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