Ekonomia. Rynek, gospodarka, społeczeństwo
47(2016), s. 79−121
DOI: 10.17451/eko/47/2016/210
ISSN: 0137-3056
www.ekonomia.wne.uw.edu.pl

Applying exogenous variables and regime switching to multi-factor models on equity indices*

Paweł Sakowski**, Robert Ślepaczuk**, Mateusz Wywiał**

Abstract

This article aims to extend the evaluation of classic multi-factor models of Carhart (1997) and to expand analysis performed in Sakowski, Ślepaczuk and Wywiał (2015). We test several modifications of these models to take into account different dynamics of equity excess returns between emerging and developed equity indices. Proposed extensions include volatility regime switching mechanism and three new risk factors. Additionally, we introduce common- and country-specific variables in order to control for global risk. Instead of individual stocks, we use weekly data of 81 world investable equity indices in the period of 2000−2015. We find substantial differences between results for classical models on single stocks and models evaluated for equity indices. Moreover, we observe solid discrepancies between results for developed and emerging markets. Introducing new risk factors and additional variables increase explanatory power of models.

Keywords: asset pricing models, equity risk premia, market price of risk, emerging and developed equity indices
JEL Codes: C15, G11, F30, G12, G13, G14, G15
DOI: 10.17451/eko/47/2016/210

* The support of NCN grant number 2014/13/B/HS4/03209 is gratefully acknowledged. The views presented in this text are those of the authors and do not necessarily represent those of Union Investment TFI S.A. or Quedex Derivatives Exchange

** RS (corresponding author: rslepaczuk@wne.uw.edu.pl): Faculty of Economic Sciences, University of Warsaw and Union Investment TFI S.A.; PS: Faculty of Economic Sciences, University of Warsaw; MW: Faculty of Economic Sciences, University of Warsaw and Quedex Derivatives Exchange.

1. Introduction

The estimation of equity risk premium based on multi-factor models is one of the most developed branches of financial research. Beginning with three-factor model of Fama and French (1992), who extended the classic CAPM model (Sharpe 1964; Lintner 1965; and Black, Jensen, and Scholes (1972), the research has taken two paths. The first one aims at finding a sparse set of factors, extending or replacing Fama-French factors. This includes:

- value investing strategy effects − either considered analogically to HML factor described in Fama and French (1992) (i.e., investing in stocks that have high book to market value), as included in Carhart (1997) or taking a silightly different path and employing dividends yield, earnings ratio, etc. (Lakonishok, Shleifer, and Vishny 1994; Arshanapalli, Coggin, and Doukas 1998);
- effect of low market capitalization of single stocks, found in Fama and French (1992) and repeated e.g. in Fama and French (2012);
- momentum and reversal effect, captured first by Jegadeesh and Titman (1993) and then included in numerous factor models, most significantly in Carhart (1997) four-factor model, but also in Wu (2002) and Asness (1995);
- effects of measured liquidity (Rahim and Noor 2006; Liu 2004);
- other fundamental factors, including investment factor and return on equity factor (Chen, Nowy-Marx, and Zhang 2011), or profitability factor and investment factor (Fama and French 2015), quality minus junk factor (Asness, Frazzini, and Pedersen 2013), cash-flow-to-price factor;
- innovative factors basing on the previous research results, such as measuring CAPM beta and betting against it (Frazzini and Pedersen (2014).[1]

The second direction is to implement the exisiting models in a different geographical and market setting, i.e. moving away from the spectrum of US equities. This includes:

- research on the three- and four-factor model in the context of global equities (Griffin 2002), only emerging markets (Cakici, Fabozzi, and Tan 2013), Baltic countries (Lieksnis 2010), or single country (Connor and Sehgal 2001);
- search for a different set of factors that explain variability of equities in a specific country or region, e.g. Eastern Europe (Foye, Mramor, and Pahor 2013), or in a global setting (Hou, Karolyi, and Kho 2011).

Naturally, these two paths were not mutually exclusive and research often overlaps both of them. It is also the case of our research, initiated by Sakowski, Ślepaczuk, and Wywiał (2015), where we propose a shift from single stocks

[1] The Betting Against Beta (or shortly BAB) factor captures the phenomenon that long leveraged low beta assets and short deleveraged to 1 high-beta assets produce significant positive risk-adjusted returns.

to equity indices, one for each country, to reflect the point of view of a global investor allocating his portfolio between different countries. Apart from application of Carhart (1997) in this setting, we move to include extensions to the initial model, by searching for additional factors such as realized volatility risk premium factor constructed in Sakowski, Ślepaczuk, and Wywiał (2016a) or inclusion of dynamic aspects, most notably Markov regime-switching model with two volatility regimes (Sakowski, Ślepaczuk, and Wywiał 2016a). This paper continues this work, retaining dynamic extensions and focusing on multi-factor models on equity indices, but also extends it with a search for additional factors as well as external regressors.

It is worth mentioning that there is an interesting contribution from Donadelli and Prosperi (2011), who analyse the differences in behavior of equity risk premia in emerging versus developed countries. They employ a similar aspect as our research, namely, representing each country with an equity index, initially as calculated by MSCI. They analyze statistical properties of ERP, as well as develop a simple CAPM model using those indices, with Fama and French market risk factor as well as MSCI World as a proxy for market risk, and find that both parameters estimates and the intercept are significant and, in addition to that, the intercept is always positive. In the next step, the authors changed the representing index to the one calculated using S&P/IFCI methodology, which selects stocks based on the size, liquidity and industry, thereby including some liquidity component. The authors also extend the analysis with an inclusion of trade integration and local liquidity, arriving at the conclusion that financial integration and liquidity flowing from the global market (with its cyclical behavior) explain significant portion of ERP variability.

Our research intend to complement the research already present and reviewed above, by stating and answering the following questions:

1. Can multi-factor models be used for explanation of equity risk premium for global indices?
2. Can we distinguish any common or country specific macroeconomic variables which increase explanatory power of our multi-factor models?
3. Are sensitivities to risk factors stable across countries? Do they differ during various phases of economic cycles?
4. Can we include volatility risk factor, percentage deviation from nominal GDP or relative small minus big factor to better explain variability of risk premia?
5. Does volatility regime switching mechanism enable us to explain equity risk premium for global indices in the extended models as well as in the case of five-factor model presented in Sakowski, Ślepaczuk, and Wywiał (2016a)?

The structure of the paper is as follows: section 2 presents methodology of our study. Equity risk premium, functional forms of the alternative models and econometric issues are discussed in this part. Section 3 provides description of both the data and the procedure we used to build risk factors and external regressors. We also analyse dynamics in time of risk factors and macroeconomic variables in time here. Section 4 presents results. The last section concludes.

2. Methodology

2.1. Motivation

The methodology is based on the seminal paper of Carhart (1997), who proposed the four-factor model for analysis of mutual funds performance. One of the reasons why we prefer the model of Carhart (1997) over the methodology of Fama and French (1992) (the three-factor model for stock return analysis) are the results of Fama and French (2012) and comprehensive results obtained for emerging markets by Cakici, Fabozzi, and Tan (2013). They focused on 18 emerging markets treating each of them separately. Their results revealed significance of value and momentum everywhere except Eastern Europe and additionally showed that momentum and value factors were negatively correlated.

At this moment it is important to explain rationale for choosing equity indices instead of single stocks. The main reason behind this is that from the global investment perspective single countries may be treated as an asset class. This issue is very important from the global portfolio selection problem, where asset allocation approach seems to gain more popularity. This is confirmed by the dynamic development of ETFs and derivatives providing country exposure. This approach seems to better reveal global factors than regressions on single stocks and enables equity risk premia for countries to be assessed separately. What is also important, the literature on this subject is currently very limited.

Taking into account that our research is intended for equity indices with special focus on emerging and developed markets, we propose several amendments to the initial methodology of Carhart (1997). Neccessary modifications include:

- converting monthly to weekly data in order to reveal dynamics during shorter time intervals;
- including new risk factors that explain the diversity of returns more deeply, i.e. realized volatility as the fifth factor;
- necessary conversion of well-known risk factors from the single country level to the worldwide level, including currency conversion;
- creating an adequate zero investment portfolio that fully reflects the influence of particular risk factor on equity risk premia;
- introducing volatility switching mechanism to take into account different dynamics of equity indices during high and low volatility periods;
- inclusion of common and country specific variables to multi-factor regressions in order to reveal various risks affecting mainly emerging market economies.

2.2. Equity risk premium

It is also important to define the equity risk premium as the expected excessreturn of equities over the risk free rate. The point here is that current literature (Duarte *et al.* 2015) proposes many alternative ways to measure it. Equity risk premium can be measured using:

- historical returns approach:

$$ERP = \frac{1}{N}\sum_{t=t_0}^{N} R_t - Rf_t \tag{1}$$

where $R_t - Rf_t$ is the return at time *t* over the risk-free rate;

- earnings yield approach:

$$ERP = \frac{E}{P} - R_f \tag{2}$$

where $\frac{E}{P}$ is earning to price ratio;

- dividend yield approach

$$ERP = \frac{D}{P} + g - R_f \tag{3}$$

where $\frac{D}{P}$ is dividend to price ratio and *g* is dividend growth rate;

- regression- and factor-based approach which can be characterized by point-in-time estimates instead of long-term estimates only, not dependent on e.g. tax policy, and which allows for dynamic forecasts:

$$ERP_t = \alpha + \sum_{i=1}^{n} \beta_i X_{i,t} + \epsilon_t \tag{4}$$

where $X_{i,t}$ is the *i*-th risk factor at moment *t* and β_i is the sensitivity to this factor;

- survey-based approach, which may often produce systematically biased estimates, negatively correlated with future returns, and positively with previous returns.

In this article, when we talk about equity risk premium, we refer to the ERP measure using the factor-based approach. Choice of the particular way of measurement of equity risk premium can certainly affect the final conclusions. Before we focus on this issue, we describe alternative factor models used in this research.

2.3. Multi-factor models

2.3.1. Initial model

We start with the classic Carhart four-factor model:

$$(R_i - R_f)_t = \alpha_i + \beta_{MKT,i} \times (R_m - R_f)_t + \beta_{HML,i} \times HML_t + \beta_{SMB,i} \times SMB_t + \beta_{WML,i} \times WML_t + \varepsilon_{i,t} \tag{5}$$

where: $(R_i - R_f)_t$ is the weekly return of equity index in excess to the weekly free rate; $(R_m - R_f)_t$ is the equally weighted equity index less the risk free rate; HML_t is the weekly premium on the book-to-market factor; SMB_t is the weekly premium on the size factor; WML_t is the weekly premium on winners-minus-losers factor; ε_t is the error term.

The WML factor is calculated by subtracting the equal weighted average of the highest performing equity indices from the equal weighed average of the lowest performing equity indices (Carhart 1997).

Next, we add to the model an additional factor based on realized volatility (VMC − volatile minus calm):

$$(R_i - R_f)_{i,t} = \alpha + \beta_{MKT,i}(R_m - R_f)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t + \varepsilon_{i,t} \tag{6}$$

The VMC factor is the weekly premium on volatile minus calm (VMC) equity indices and is obtained by subtracting the equal weighted average return of the highest volatility equity indices from the equal weighted average return of the lowest volatility equity indices. The definition of high or low volatility is based on 63 days realized volatility calculated separately for each equity index.

The detailed procedure for calculating HML, SMB, VML and VMC risk factors and definitions of zero-investment portfolios based on them is summarized in Section 3.1. The VMC factor was introduced in Sakowski, Ślepaczuk, and Wywiał (2016a) and the details and rationale for this factor have been given there.

Since we want to take into account an impact of different market environments (that is, low-volatility periods, including moderate bull markets or horizontal trends, and high-volatility trading conditions, such as downturns) on the factor sensitivities, we consider adding to the model a regime switching mechanism. The first attempt to capture a change in the volatility regime focuses on including different dynamics of equity risk premia: (1) in high and low volatility environment, and (2) during upward and downward movements of the market. We add dummy variables with appropriate interactions and the functional form of the regression is given by:

$$(R_i - R_f)_{i,t} = \alpha_i + \beta_{MKT,i}(R_m - R_f)_t + \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t + \gamma_i D_t + \gamma_{MKT,i}(R_m - R_f)_t D_t + \gamma_{HML,i}HML_t D_t + \gamma_{SMB,i}SMB_t D_t + \gamma_{WML,i}WML_t D_t + \gamma_{VMC,i}VMC_t D_t + \varepsilon_{i,t} \tag{7}$$

We consider two alternative definitions of the dummy variable used in equation (7):

- $D_t = 1$ for high volatility periods, $D_t = 0$ for low volatility periods, where the division is based on realized volatility calculated in USD for the market index and the period brackets were defined *ex-ante*;

- *ex-post* identification of upward ($D_t = 1$) and downward ($D_t = 0$) movements of the market.[2] Such division is especially important when we perform analysis between 2000 and 2015, since this period of equity markets was characterized by two strong bull and two strong bear markets which was not observable before within such relatively short data span.

The last form of the multi-factor model tested in the first part of this paper is a simple form of Markov switching model with two states of the world. For this purpose we used five-factor model in the form presented in equation (6). The rationale for the selection of the last model was based on several studies which showed, among other things, that various types of regime switching models can be useful in explaining equity risk premia (Tan 2013). Ammann and Verhofen (2006) revealed that value investing seems to be a rational strategy in the High-VarianceRegime, while momentum investing in the Low-Variance Regime. They additionally presented an empirical out-of-sample backtest indicating that this switching strategy can be profitable. Moreover, Angelidis and Tessaromatis (2014) indicated that there are significant costs to investors who fail to take into account the existence of regimes in portfolio construction and asset allocation. Hammerschmid and Lohre (2014) showed that regime shifts are preserved in the presence of fundamental variables known to predict equity risk premia.

Hence, the complete list of models used in the first part of this study is presented below:

1. fac4 (four-factor model, factors based on local currency);
2. fac4.usd (four-factor model, factors based on USD);
3. fac5.usd (five-factor model, VMC added, factors based on USD);
4. fac5.usd.rv.dummy (five-factor model, VMC factor added, factors based on USD, added a dummy variable based on 3-month realized volatility threshold of 15%);
5. fac5.usd.up.down.dummy (five-factor model, VMC and market trend dummies added, factors based on USD);
6. fac5.usd.markov (five-factor model, VMC and added, factors based on USD).

These models have been evaluated in Sakowski, Ślepaczuk, and Wywiał (2016a) and are included in this paper for the purpose of further analysis and comparison. All necessary details can be found there. This paper focuses on the extensions of these models, outlined in the next subsections.

[2] In the course of our analysis, the periods of high volatility usually corresponded to the periods of downward market movements, while the low volatility periods occurred together with the moderate upward movements.

2.3.2. Extensions – additional factors and variables

Based on the initial results, we decided to develop this research and perform additional analysis in order to increase explanatory power of the models. For this purpose we defined two additional factors (QMS-positive minus negative, SMBrel-relative small minus big) and moreover we added common and country specific variables which then will be used in the multi-factor model. After this step, the general functional form of multi-factor model has the following shape:

$$\begin{aligned}(R_i - R_f)_{i,t} &= \alpha + \beta_{CV,i}CV_t + \beta_{CspV,i}CspV_t + \beta_{MKT,i}(R_m - R_f)_t \\ &+ \beta_{HML,i}HML_t + \beta_{SMB,i}SMB_t + \beta_{WML,i}WML_t + \beta_{VMC,i}VMC_t \\ &+ \beta_{QMS,i}QMS_t + \beta_{SMBrel,i}SMBrel_t + \varepsilon_{i,t}\end{aligned} \tag{8}$$

The QMS factor is the weekly premium on positive minus negative (QMS) equity indices and is obtained by subtracting the equal weighted average return of equity indices characterized by the highest positive percentage deviation of nominal GDP from its 5y moving average and the equal weighted average return of the equity indices characterized by the lowest positive percentage deviation of nominal GDP from its 5y moving average.

The SMBrel factor is similar to SMB factor with one important exception. The sorting of equity indices to each decile group is based on the ratio of index capitalization to GDP of the given country instead of capitalization only. Such approach enables us to refer in this factor to the level of capital market development and not only to absolute value of capitalization. This is due to the fact that, in reality, the absolute values of market capitalization are not straightforwardly comparable among countries.

The detailed procedure of calculating QMS and SMBrel risk factors and definitions of zero-investment portfolios based on them are summarized in Section 3.1.

Common variables (denoted in the formula above) are the same for all equity indices under investigation. The list of common variables will be added to our regression as follows:

1. GSCI − S&P Goldman Sachs Commodity Index. This variable represents main trends on aggregated commodity prices. We use weekly differences of this index. The rationale for this variable comes from the substantial dependence among commodity dependent countries (producers and importers) and the prices of commodities.
2. SKEW − skew of S&P500 index options. We use weekly differences of 4 weeks moving average of this index. This variable can be very important in signalling market crashes.
3. OVXGVZ − the average of annualized monthly RV of crude oil prices and GVZ volatility indices. We use weekly differences of average of these indices. These volatility indices are a good proxy of the sentiment of oil and gold investors.

4. VX – the average of VIX, VXEEM, VXEWZ and VXFXI volatility indices. We use weekly differences of average of these indices. These volatility indices are a good proxy of the sentiment of investors focused on the US market, emerging markets, Brazil and China.
5. DXY − USD index. We use weekly differences of this index. The rationale for this variable is its high negative correlation with commodity prices which are mainly priced in USD.
6. CLF_dummy − dummy variable which has value of 1 when crude oil price is lower then 65 USD and 0 otherwise.
7. VVIX − volatility of VIX index, i.e. volatility of volatility of SPX index. We use weekly differences of this index. The rationale for this variable is its very high correlation with equity market crashes.

Country-specific variables (denoted in the equation above) are aggregated into two groups, separately for developed and emerging countries. They are built based on rates and currency fluctuations:

1. STdev − short-term rates for developed countries. This variable is built based on single short-term rates for developed countries. It is the moving average of weekly returns from ST rates.
2. STem – short-term rates for emerging countries. This variable is built based on single short-term rates for emerging countries. It is the moving average ofweekly returns from ST rates.
3. LTdev – long-term rates for developed countries. This variable is built based on single long-term rates for developed countries. It is the moving average of weekly returns from LT rates.
4. LTem − long-term rates for emerging countries. This variable is built based on single long-term rates for emerging countries. It is the moving average of weekly returns from LT rates.
5. Cdev − currencies for developed countries. This variable is built based on single currency rate fluctuations for developed countries. It is the moving average of weekly returns from currency rates.
6. Cem − currencies for emerging countries. This variable is built based on single currency rate fluctuations for emerging countries. It is the moving average of weekly returns from currency rates.

Hence, the complete list of models used in the second part of this study is presented below:

1. fac5.usd (five-factor model, including VMC, factors based on USD);
2. fac7.usd (seven-factor model, VMC, QMS and SMBrel added, factors based on USD);
3. fac5.cs.usd (five-factor model, VMC and country-specific variables added, factors based on USD);

4. fac5.common.usd (five-factor model, VMC and common variables added, factors based on USD);
5. fac5.common.cs.usd (five-factor model, including VMC, common and country-specific variables added, factors based on USD);
6. fac7.common.cs.usd (seven-factor model, VMC, QMS, SMBrel, common and country-specific variables added, factors based on USD).

2.3.3. Switching mechanism

Based on the results presented in Sakowski, Ślepaczuk, and Wywiał (2016a) and preliminary ones from sections 2.3.1 and 2.3.2, we decided to test Markov switching model with two states of the world on the same set of variables as presented in equation (8). Our intuition was that adding new set of variables to Markov switching models could help us explain substantial differences in explanatory power of our models between emerging and developed markets. The estimation procedure of each Markov switching model is the same as in Sakowski, Ślepaczuk, and Wywiał (2016a). In particular, the Markov switching was applied to the parameters next to each variable included in the respective model (this includes the intercept).

Hence, the complete list of models used in the third part of this study is presented below:

1. fac7.common.cs.msm.usd (seven-factor Markov switching model, VMC, QMS, SMBrel, common and country-specific variables added, factors based onUSD);
2. fac7.common.msm.usd (seven-factor Markov switching model, VMC, QMS, SMBrel and common variables added, factors based on USD);
3. fac7.cs.msm.usd (seven-factor Markov switching model, VMC, QMS, SMBrel and country-specific added, factors based on USD);
4. fac5.common.cs.msm.usd (five-factor Markov switching model, VMC, QMS, SMBrel, common and country-specific variables added, factors based on USD);
5. fac5.common.msm.usd (five-factor Markov switching model, VMC, QMS, SMBrel and common variables added, factors based on USD);
6. fac5.cs.msm.usd (five-factor Markov switching model, VMC, QMS, SMBrel and country-specific added, factors based on USD);
7. fac5.msm.usd (five-factor Markov switching model, Carhart+VMC, factors based on USD).

2.4. Methodological and diagnostic issues

In the process of estimation of the multi-factor models using time-series data, we can potentially encounter several econometric problems or issues which should be resolved in the process of estimation (possible ARCH effect, autocorrelation, heteroscedasticity of the error term or differences between various methods of estimation of the models). Surprisingly, this issue is barely ever discussed in the literature of multi-factor models.

An attempt to estimate our regressions correctly with full econometric diagnostics takes us to the point where we should choose one of the paths below:

1. To estimate models with the same functional forms and compare their results across all markets, ignoring any diagnostic issues, as it has been presented in financial literature for years.
2. To perform all diagnostics concerning time-series issues and correct the first estimations, which will most probably result in different model functional forms across investigated markets and hence make it difficult to compare the results for them.

Taking into account that we do intend to compare single alpha, beta coefficients and R squared coefficient among equity indices, we decided to choose the first approach. We believe that this allows us to analyze explaining power of models estimated for different markets. All in all, the issue of performing model diagnostics seems to be important and we decided to devote to this issue separate paper Sakowski, Ślepaczuk, and Wywiał (2016b).

3. Data

We gathered the data for the most comprehensive set of investable equity indices[3] covering the period between 1990 and 2015. However, the study was intentionally limited to 2000−2015 because of unavailability of longer time series for some of the risk factors, especially for emerging market countries. The data set, including country list, was more thoroughly described in our previous papers, including Sakowski, Ślepaczuk, and Wywiał (2016b).

The analysis was performed on weekly data for 81 most representative and investable equity indices, covering all continents. We included data of indices for 27 developed and 54 emerging markets. The detailed list of all equity indices, risk factors and common and country specific variables and their descriptive statistics can be obtained upon request.

The reason behind selection of weekly instead of monthly data was to evaluate theoretical value of excess returns for the given equity index more frequently.

[3] For practical purposes we used only these indices which can be easily invested through options, futures or ETFs.

All returns and risk factors, with exception of the four-factor model of Carhart with factors based on local currency (fac4), were calculated after converting local prices to USD. Surprisingly however, results did not differ significantly between the same model calculated in local currency and in USD.

3.1. Description of risk factors

Analysis of dynamics of standard four factors from the Carhart model helped us to define the final specification of the five factor model. Below we present the detailed description of procedure of calculating HML, SMB, VML, VMC, QMS and SMBrel risk factors, definitions of zero-investment portfolios based on them and then our observation concerning dynamics of these factors. Each factor was obtained by investing in relevant decile portfolios calculated for each moment of time, so that the factor returns do not have any look-ahead bias.

The ($R_m - R_f$) factor represents weekly excess return of the market portfolio over the risk-free rate. The market portfolio consists of 81 equity indices which are equally weighted. The HML is a zero-investment portfolio that is long on the highest decile group of book-to-market (B/M) equity indices and short on the lowest decile group. The difference in returns of these extreme decile groups is calculated in each weekly interval, which finally constitutes HML factor. Based on these returns we created cumulative returns for HML and then LMH zero investment portfolio.

The SMB is a zero-investment portfolio that is long on the highest decile group of small capitalization (cap) equity indices and short on the lowest decile group. The difference in returns of these extreme decile groups is calculated in weekly interval as well. Similarly, based on these returns we created cumulative returns for SMB and then BMS zero investment portfolio.

The WML is a zero-investment portfolio that is long on the highest decile group of previous 1-year return winner equity indices and short on its lowest decile group (loser equity indices). The difference in returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for WML and then LMH zero investment portfolio.

The VMC is a zero-investment portfolio that is long on the highest decile group of high volatility equity indices and short on its lowest decile group (low volatility equity indices). The difference in returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for VMC and then CMV zero investment portfolio.

The QMS factor is a zero-investment portfolio that is long on the highest decile group equity indices characterized by the highest positive percentage deviation of nominal GDP from its 5-year moving average and short on its lowest decile group (the highest negative percentage deviation of nominal GDP from its trend). The difference in returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for QMS and then SMQ zero investment portfolio.

Finally, the SMBrel is a zero-investment portfolio that is long on the highest decile group of equity indices characterized by the lowest ratio of cap to nominal GDP and short on its low highest decile group. The difference in returns of these extreme decile groups is calculated again for each weekly interval and based on that we create cumulative returns for SMBrel and then BMSrel zero investment portfolio.

3.2. Analysis of risk factors' dynamics, common variables and country specific variables

3.2.1. Risk factors

Figure 1 shows the dynamics of the market index factor (R_m-R_f) and of market index returns (R_m). We cannot observe any substantial differences between them. This actually informs us that we analyzed the period of exceptionally low rates, and that interest rates had only marginal impact on the value of this factor.

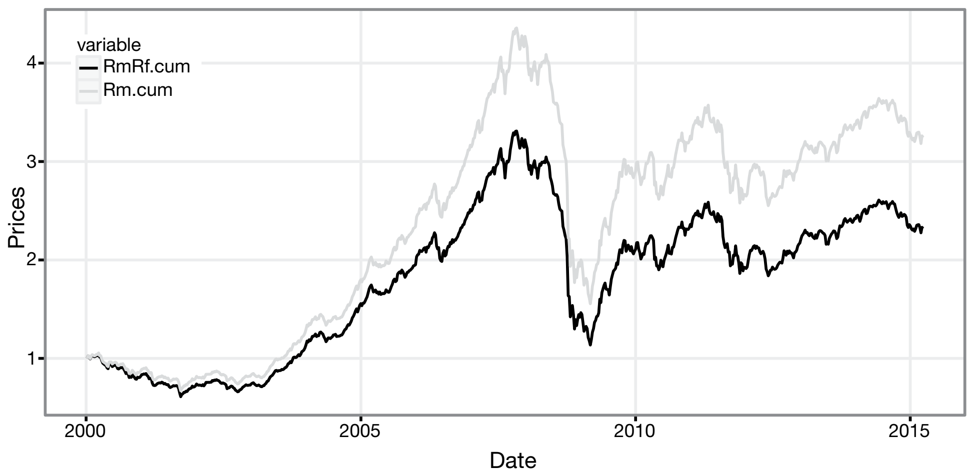

Figure 1. Dynamics of cumulative R_m-R_f factor and separately for market index (R_m)

Note: R_m-R_f factor was calculated on weekly data in USD between 2000−2015. R_m represents equally weighted market index based on USD. Lines present cumulative returns for R_m-R_f and R_m factors.

Source: authors' own calculations.

Figure 2 presents fluctuations of the second factor (HML_t). It reveals two distinct periods. The first one (2000−2011) shows a strong HML effect showing much better performance of equity indices with high book-to-market characteristics. Similar phenomenon has been heavily presented in the literature for stock returns. However, in the second period, starting from 2012, the HML effect disappeared and has been entirely transformed into the LMH effect which is quite surprising and requires additional research.

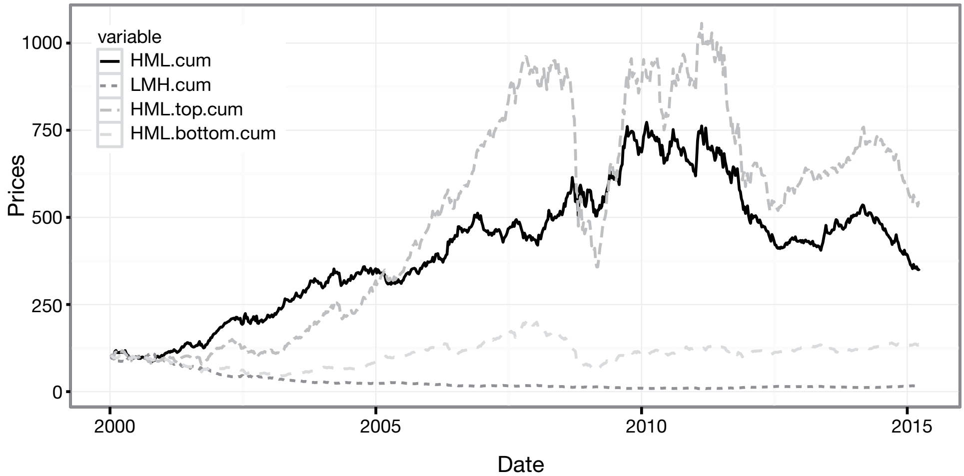

Figure 2. Cumulative returns of HML factor with top/bottom 10% percentiles

Note: HML factor was calculated on weekly data between 2000−2015. Lines present cumulative returns for HML, LMH, top and bottom book-to-market values decile portfolios, respectively.

Source: authors' own calculations.

Fluctuations of the third risk factor (SMB_t) are presented in the Figure 3. Again, it can be divided into two periods. The first one, between 2000−2008, is characterized by outperformance of small capitalization equity indices what was revealed in the literature for single stocks. In the second period (2008−2015), this effect is totally reversed and we can observe outperformance of big capitalization equity indices.

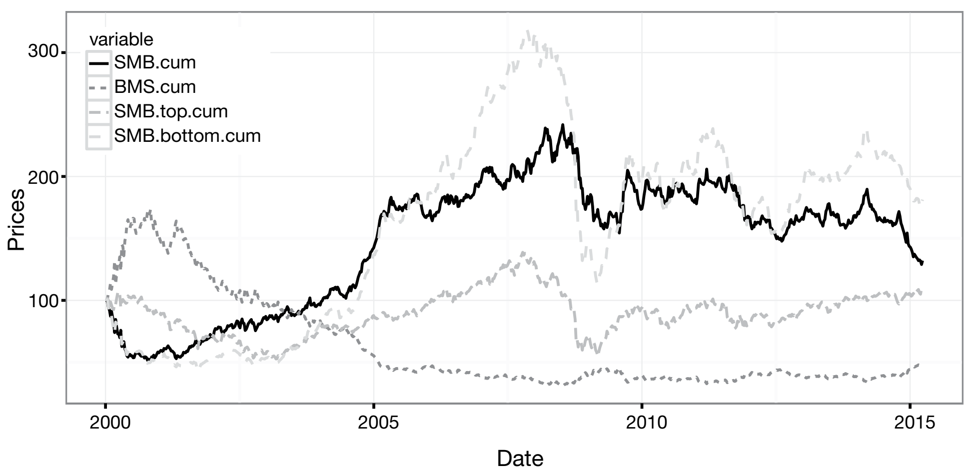

Figure 3. Cumulative returns of SMB factor with top/bottom 10% percentiles

Note: SMB factor was calculated on weekly data in USD between 2000−2015. Lines present cumulative returns for SMB, BMS, top and bottom capitalization decile portfolios, respectively.

Source: authors' own calculations.

The fourth risk factor (WML_t) shows that WML effect is the strongest one (Figure 4) and that it is quite stable during the whole period and exactly confirms the short-term momentum effect observed in financial literature.

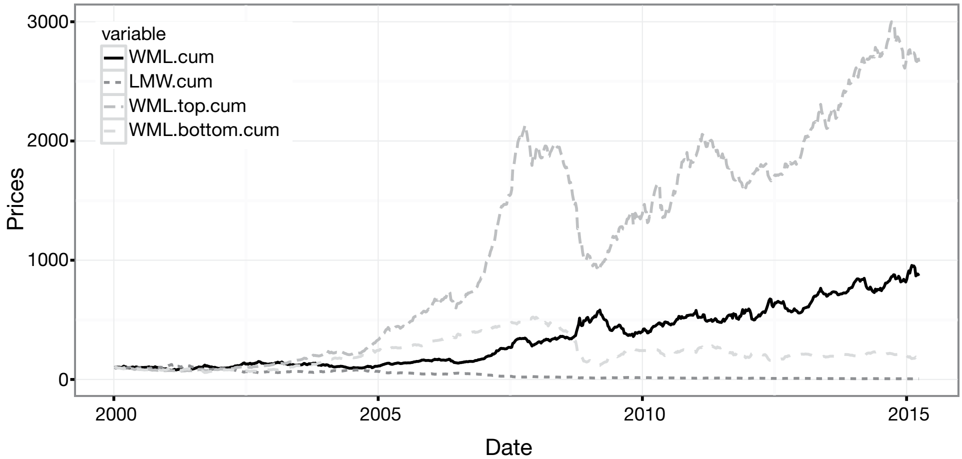

Figure 4. Cumulative returns of WML factor with top/bottom 10% percentiles

Note: WML factor was calculated on weekly data in USD between 2000−2015 with returns based on last 1 year. Lines present cumulative returns for WML, LMW, top and bottom momentum decile portfolios, respectively.

Source: authors' own calculations.

The fifth risk factor (VMC_t) reveals similar dynamics to HML and SMB effects (Figure 5) dividing the period into two different sub-periods. The first one ends exactly before the bear market in 2008 and is characterized by outperformance of high volatility equity indices. In the second period (2008−2015) this effect is exactly reversed and we can observe outperformance of low volatility equity indices.

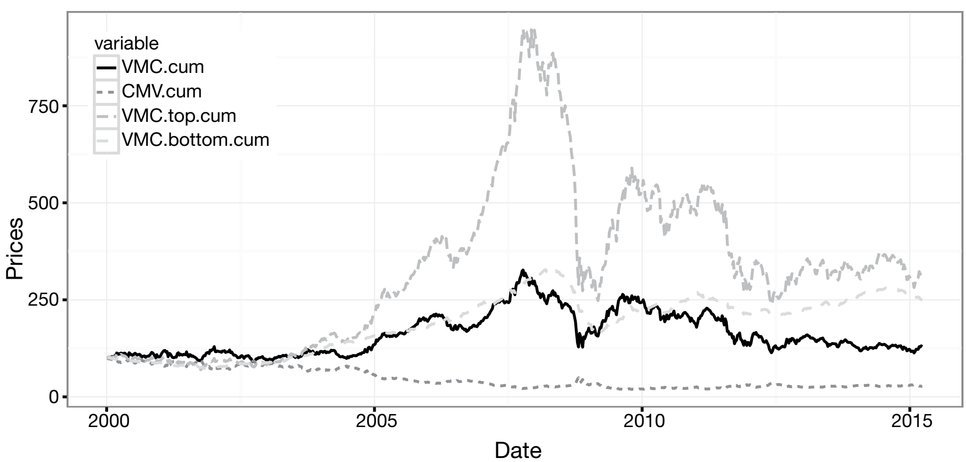

Figure 5. Cumulative returns of VMC factor with top/bottom 10% percentiles

Note: VMC factor was calculated on weekly data in USD between 2000−2015 with returns based on last 1 year. Lines present cumulative returns for VMC, CMV, top and bottom 63 days realized volatility decile portfolios, respectively.

Source: authors' own calculations.

The sixth risk factor (QMS_t) built based on percentage deviation from nominal GDP reveals once again similar dynamics to HML and SMB effects (Figure 6) and divides the period into two sub-periods. The first one showing better behavior of QMS portfolios lasted from 2000 to 2003, while the second one characterized by the better behavior of SMQ portfolios was much longer and lasted from 2003 until 2015.

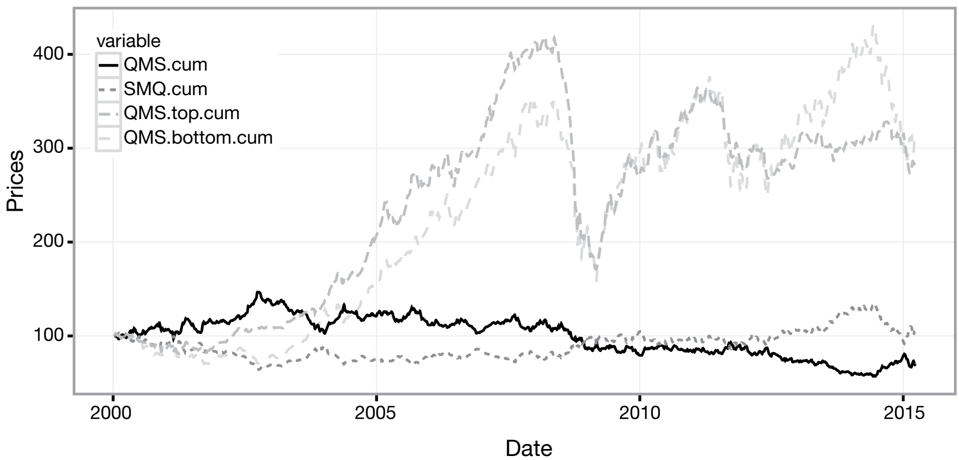

Figure 6. Cumulative returns of QMS factor with top/bottom 10% percentiles

Note: QMS factor was calculated on weekly data in USD between 2000–2015. Lines present cumulative returns for QMS, SMQ, top and bottom decile portfolios, respectively.

Source: authors' own calculations.

The seventh risk factor ($SMBrel_t$) reveals completely different dynamics in comparison to quite similar SMB factor (Figure 7). Once again we can distinguish two separate periods. This time, the analysed time series, BMSrel, exhibits positive performance in the first period between 2000 and 2008, while SMBrel shows significant uptrend in the second period, ending in 2015.

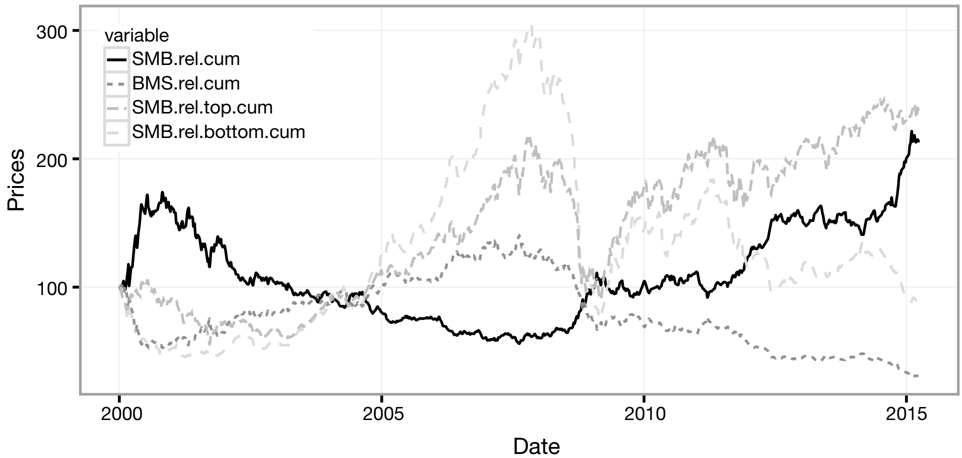

Figure 7. Cumulative returns of SMBrel factor with top/bottom 10% percentiles

Note: SMBrel factor was calculated on weekly data in USD between 2000−2015. Lines present cumulative returns for SMBrel, BMSrel, top and bottomdecile portfolios, respectively.

Source: authors' own calculations.

Presented dynamics of seven factors suggest that their explanatory power with respect to excess returns can be rather limited with the exception of the first factor. What is important is that the analysis of fluctuations of portfolios based on the risk factors in USD in comparison to their dynamics in local currency (Sakowski, Ślepaczuk, and Wywiał 2015) reveals very similar dynamics. This informs us that currency effect is not the main driver which can be used in order to explain these effects.

3.2.2. Common variables

Before we go to results section it is important to analyze the fluctuations of the new set of variables, namely common variables, which are the same for all equity indices under investigation. First one is S&PGSCI (Goldman Sachs Commodity Index) which serves as a benchmark for investment in the commodity markets and as a measure of commodity performance over time. The performance of this index in the period 1990−2015 can be found in Figure 8.

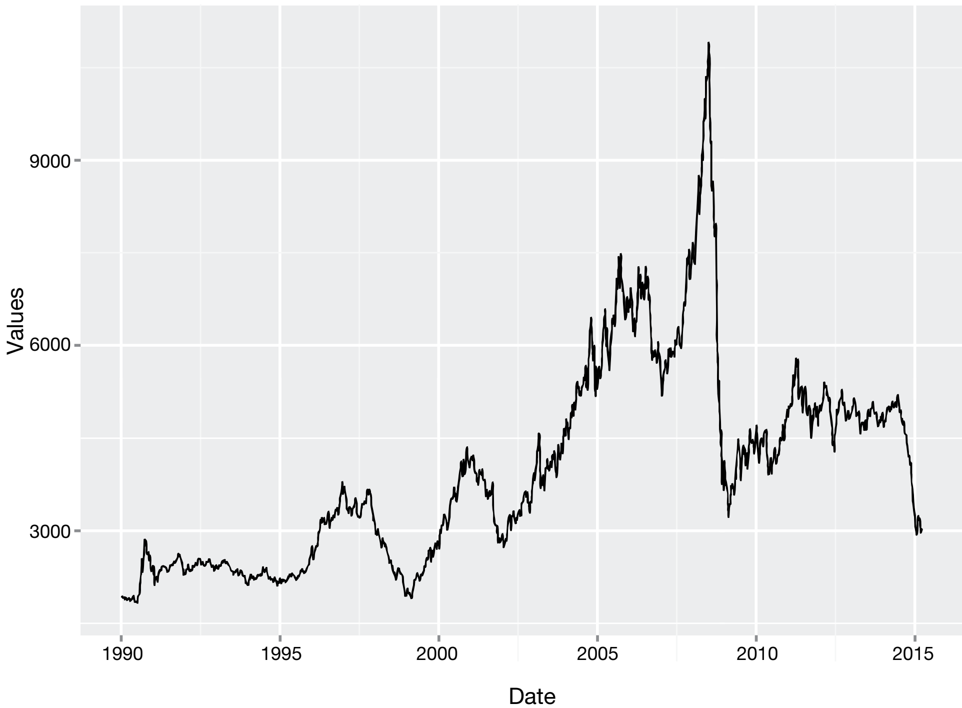

Figure 8. S&PGSCI fluctuations

Note: SPGCSI was calculated on weekly data in USD between 1990−2015.

Source: authors' own calculations.

We can distinguish very strong bull market which lasted from the beginning of 1990s until 2008 and then very strong bear market which is present on commodity markets until the end of our data set. The inclusion of this variable in our model was dictated by its strong correlation with the emerging markets.

The next variable is CBOE SKEW Index. The CBOE SKEW Index is an index[4] derived from the price of S&P 500 tail risk. Similar to VIX, the price of S&P 500 tail risk is calculated from the prices of S&P 500 OTM options. SKEW typically ranges from 100 to 150. A SKEW value of 100 means that the perceived distribution of S&P 500 log-returns is normal, and the probability of outlier returns is therefore negligible. As SKEW rises above 100, the left tail of the S&P 500 distribution acquires more weight, and the probabilities of outlier returns become more significant. The fluctuations of SKEW can be found in Figure 9.

[4] The definition was found on CBOE website: http://www.cboe.com/micro/skew/introduction.aspx.

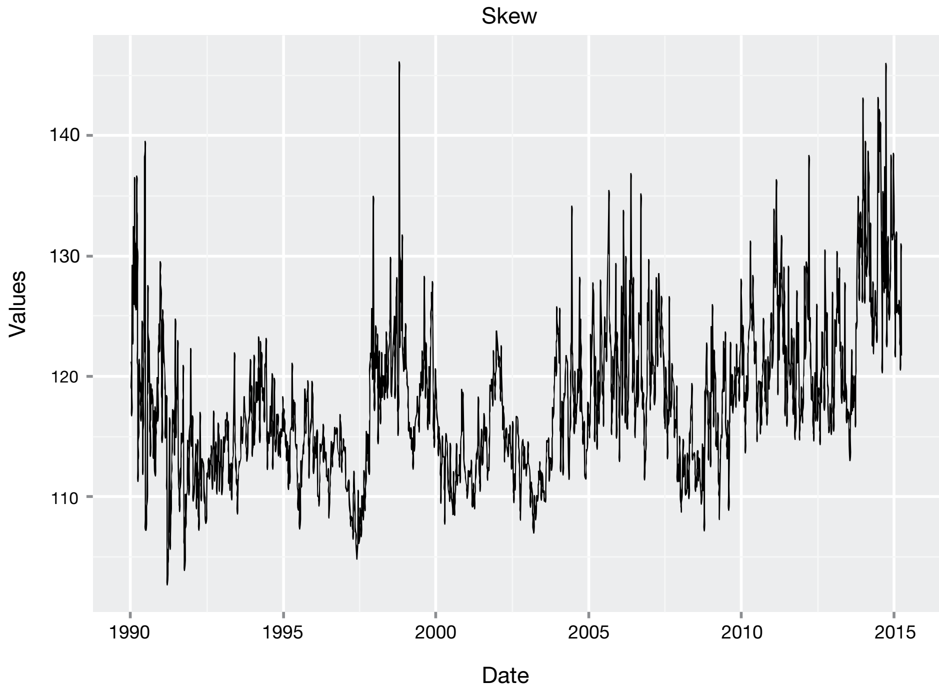

Figure 9. CBOE SKEW Index fluctuations

Note: CBOE SKEW is presented on weekly data between 1990–2015.

Source: authors' own calculations.

We can see that this measure most often ranges between 110 and 130 confirming that distribution of returns of S&P500 index rather departs from normal distribution. We included this variable in our models in order to be able to refer to high stress periods on capital markets.

The subsequent variable used in our new seven-factor model is OVXGVZ, which is the average of annualized monthly RV of crude oil prices and GVZ volatility index published by CBOE based on the same methodology like for VIX.[5] Our intention was to connect volatility of two main commodities (crude oil and gold) in one index in order to reflect their connection with emerging market economies. The behavior of OVXGVZ index is presented on Figure 10.

[5] The definition of GVZ index can be found on the following CBOE website: http://www.cboe.com/micro/gvz/introduction.aspx.

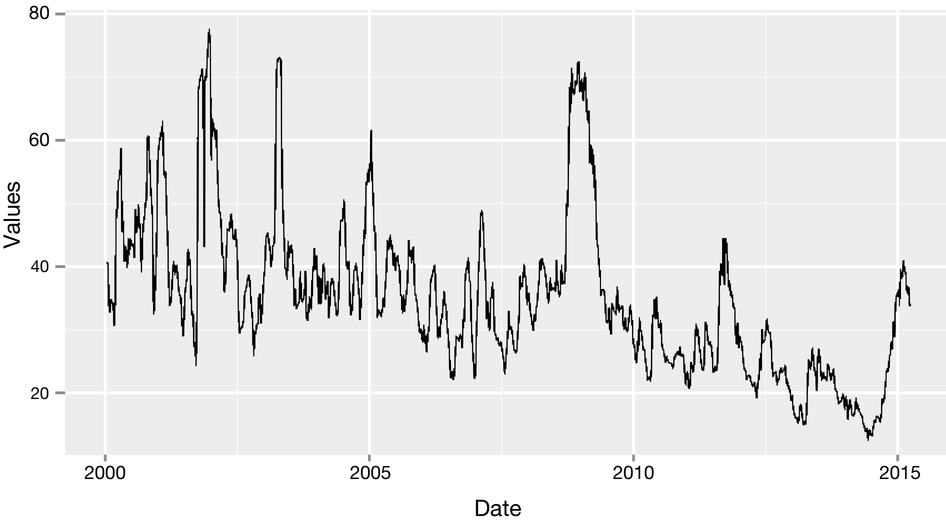

Figure 10. OVXGVZ Index fluctuations

Note: OVXGVZ was calculated on weekly data between 2008−2015. OVXGVZ is calculated as the average of annualized monthly RV of crude oil process and GVZ (volatility index of gold).

Source: authors' own calculations.

Figure 10 shows that the average level of volatility presented by OVXGVZ Index gradually declined during presented 15-year period but, at the same time, volatility jumps were much severe, partly due to much lower starting point.

Next variable (VX) also refers to volatility and is the average of VIX (volatility index of S&P500 index), VXEEM (volatility index of emerging markets EEM ETF, the iShares MSCI Emerging Markets Index), VXEWZ (volatility index of Brazil markets EWZ ETF) and VXFXI (volatility index of China markets FXI ETF) volatility indices published by CBOE.[6]

The fluctuations of VX variable are shown in Figure 11.

[6] The definitions of these volatility indices can be found on the following CBOE websites: http://www.cboe.com/micro/vix-options-and-futures.aspx, http://www.cboe.com/micro/vixetf/vxeem/, http://www.cboe.com/micro/vixetf/vxewz/, http://www.cboe.com/micro/vixetf/vxfxi.

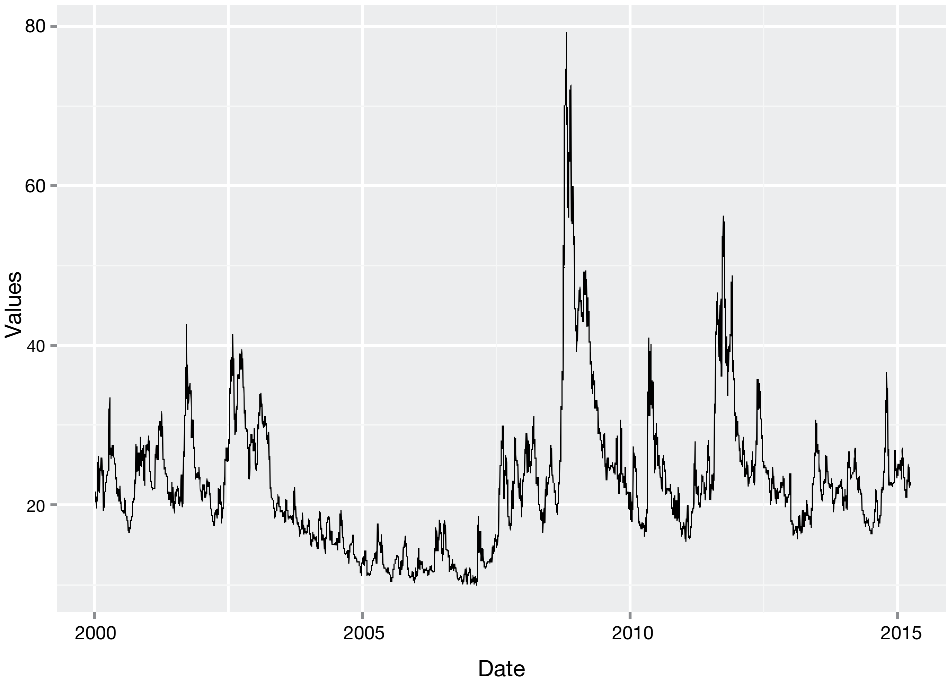

Figure 11. VX Index fluctuations

Note: VX Index was calculated on weekly data between 2011−2015.VX is calculated as an average between values of VXEEM (volatility of emerging equityindices), VXEWZ Index (volatility of brazil equity index), and VXFXI (volatility of China equity index).

Source: authors' own calculations.

We can see that 90% of fluctuations of our index ranges between 15 and 25 but it explodes even up to 80 in the time of stress on equity markets. Therefore we presume that it should be correlated with market turmoils on equity markets.

Next variable is the US Dollar Index (DXY) which is an index (or measure) of the value of the United States dollar relative to a basket of foreign currencies, often referred to as a basket of US trade partners' currencies. Once again, our main intention to add this variable to our model was to indicate the time of stress on capital markets which is usually characterized by the 'flight to hard currencies', mainly USD. The behavior of DXY is presented in Figure 12.

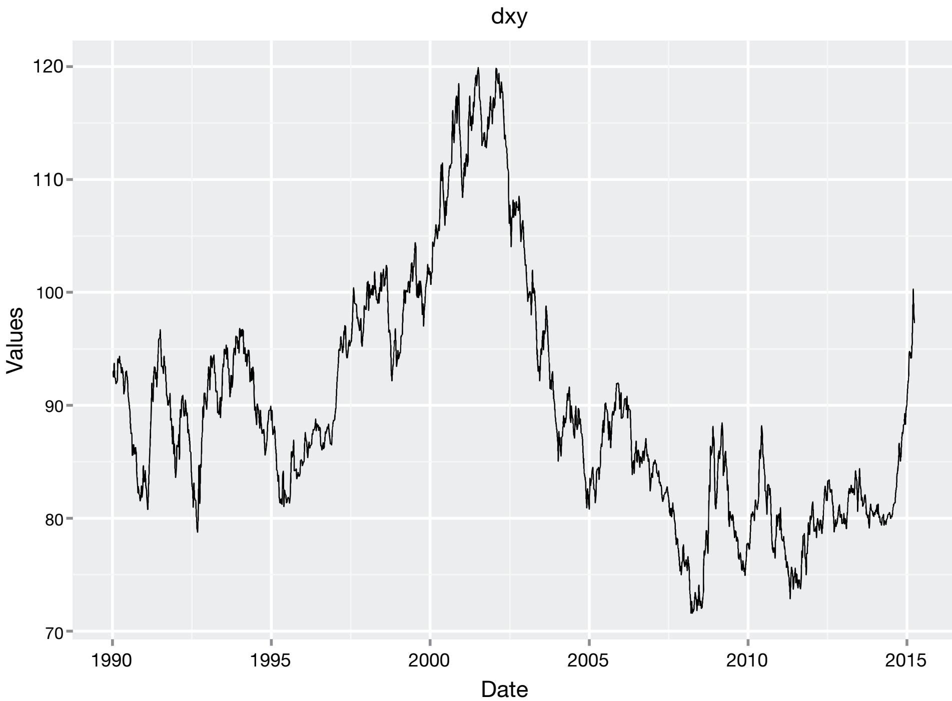

Figure 12. DXY Index fluctuations

Note: DXY Index was calculated on weekly data between 1990−2015.

Source: authors' own calculations.

DXY Index shows two strong upward trends which lasted between 1995 and 2002 (culminating at the bottom of the bear market after internet bubble) and the current one which started in 2011.

The subsequent variable (CLF_dummy) was a dummy variable which had value of 1 when crude oil price is lower than 65 USD and 0 otherwise. This variable was intentionally created as a dummy only in order to indicate period of high and low crude oil prices which are quite important for many emerging markets economies. Figure 13 shows the fluctuations of CLF_dummy.

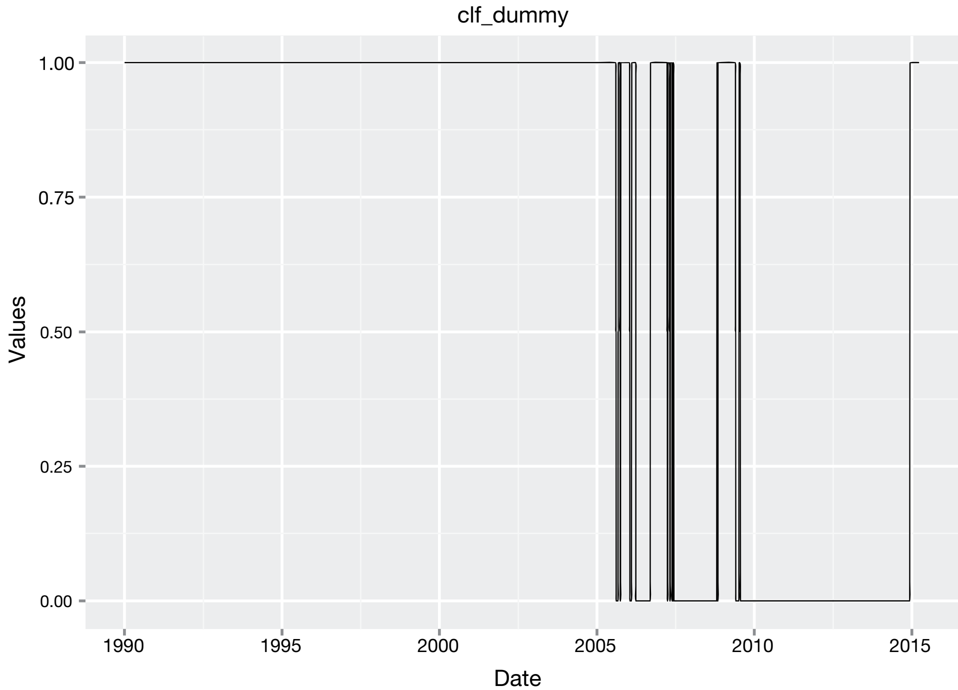

Figure 13. CLF_dummy variable fluctuations

Note: CLF_dummy was calculated on weekly data in USD between 1990−2015.

Source: authors' own calculations.

Last variable among common variables is VVIX, once again calculated by CBOE, as volatility of S&P500 index.[7] The rationale for this variable is its very high correlation with equity market crashes. We decided to use weekly differences of this index in our model. The behavior of this index can be found in Figure 14.

[7] The definition of this index can be found here: http://www.cboe.com/micro/vvix.

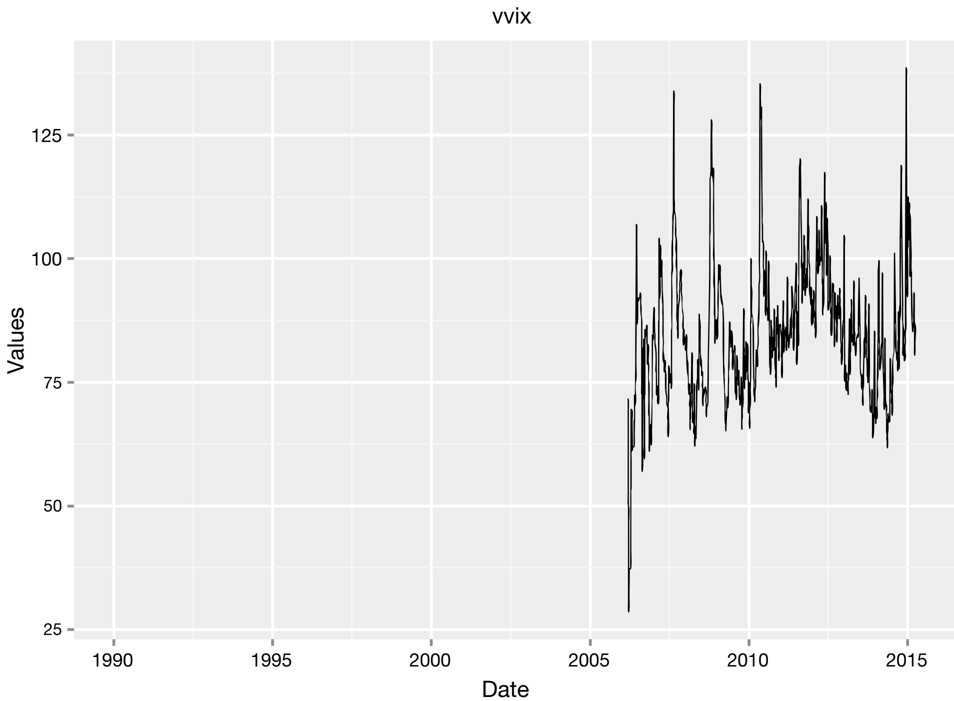

Figure 14. VVIX fluctuations

Note: VVIX was calculated on weekly data in USD between 2007−2015.

Source: authors' own calculations.

We can see that for the majority of time VVIX fluctuates between 70 and 110 and it spikes even to 200 in the time of equity market turmoils.

Generally, we can summarize that we have chosen two sets of common variables: widely connected with the behavior of emerging market economies (S&PGSCI, OVXGVZ or CLF_dummy) or the ones indicating equity or commodity market turmoils (OVXGVX, VX, SKEW or VVIX).

3.2.3. Country specific variables

The last set of variables which were introduced to the model were country specific variables, i.e. variables that were aggregated into two groups, separately for developed and for emerging countries. They were built based on rates and currency rates fluctuations.

Figure 15 presents fluctuations of short-term interest rates separately for developed and for emerging markets countries. We can see that through the whole period under investigation 'the average rate' was in downward trend with one substantial spike before the financial crisis in 2008. The average rate of short-term rates decreased in this period from 8% for emerging markets economies (4% in the case of developed ones) to 2.5% (and around 1% for developed ones).

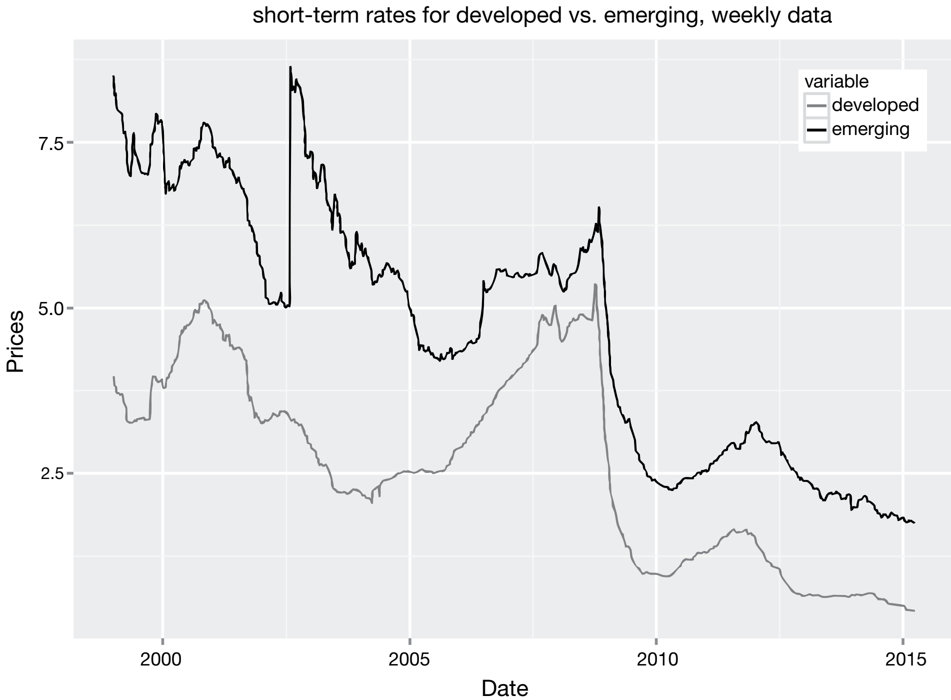

Figure 15. Short-term interest rates for developed vs. emerging market countries

Note: Short-term rates were calculated on weekly data in USD between 2000−2015. Lines present average values of short-term interest rates separately for emerging (upper line) and developed countries (lower line).
Source: authors' own calculations.

The similar situation is presented in Figure 16 where we observe long-term interest rates separately for developed and for emerging markets. The fluctuations are quite similar to short-term rates but their path downward is even more stable than in the case of long-term rates.

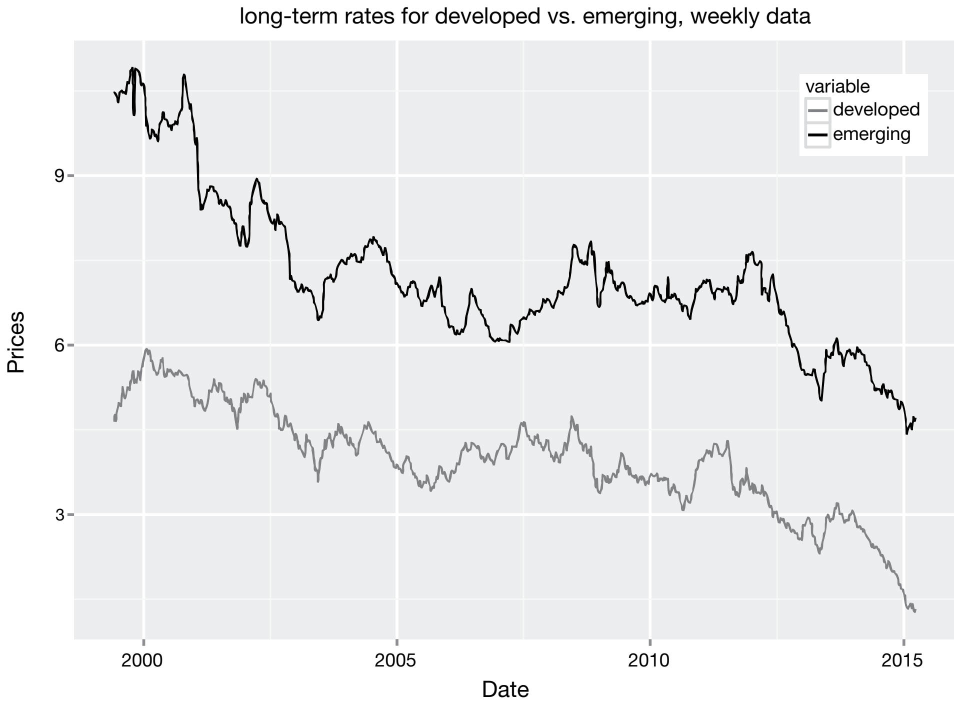

Figure 16. Long-term interest rates for developed vs. emerging countries

Note: Long-term rates were calculated on weekly data in USD between 2000−2015. Lines present average values of long-term interest rates separately foremerging (upper line) and for developed countries (lower line).

Source: authors' own calculations.

The last set of variables in the group of country specific ones are currency rates once again separately aggregated for developed and for emerging markets. Figure 17 shows their fluctuations. We can see depreciation of emerging market currencies against the USD through all the period between 1990 and 2015. Quite different situation was observed in the case of developed market currencies which depreciated against the USD over two periods: 1990−2002 and 2009−2015 and appreciated between these two periods.

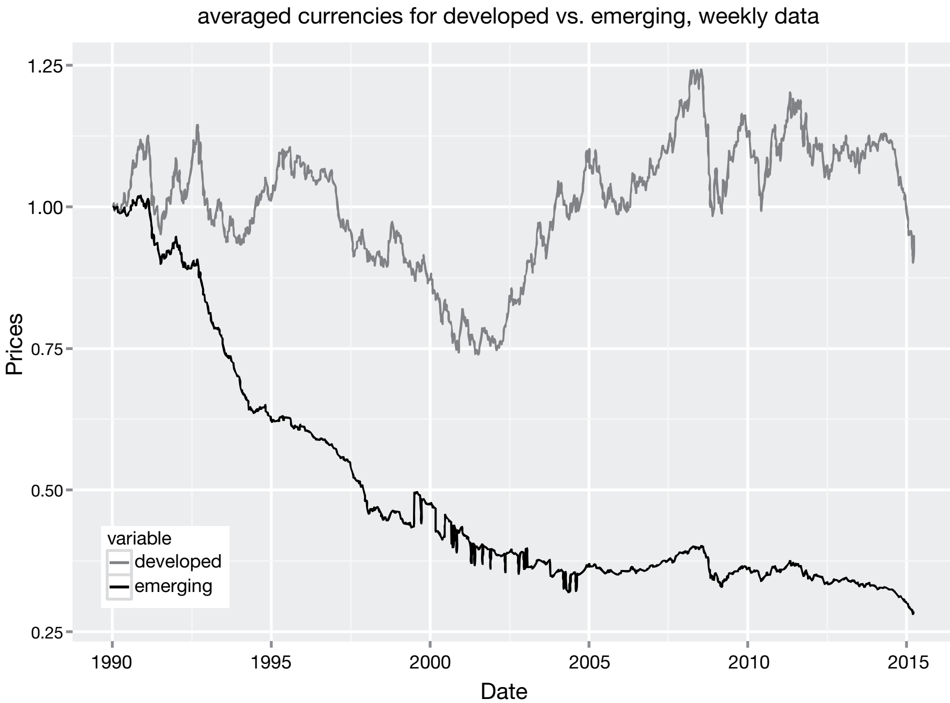

Figure 17. Currency indices for developed and emerging countries

Note: Currency indices were calculated on weekly data in USD between 2000−2015. Lines present average values of currency index separately for emerging (upper line) and for developed countries (lower line). Currency index is calculated based on average currency returns calculated separately for emerging and for developed countries.

Source: authors' own calculations.

4. Results

4.1. Comparison of various models results

4.1.1. First part of research. Variations of five-factor model

We estimated six multi-factor models from the first step of our research (six first regressions described in section 2.3.1 for 81 different countries. For clarity purposes, we start with comparison of Gaussian kernel density estimates of R-squared coefficients for all models separately for developed and for emerging markets (Figure 18). In the analysis below, every time we mention R-squared value, we refer to the adjusted R-squared coefficient.

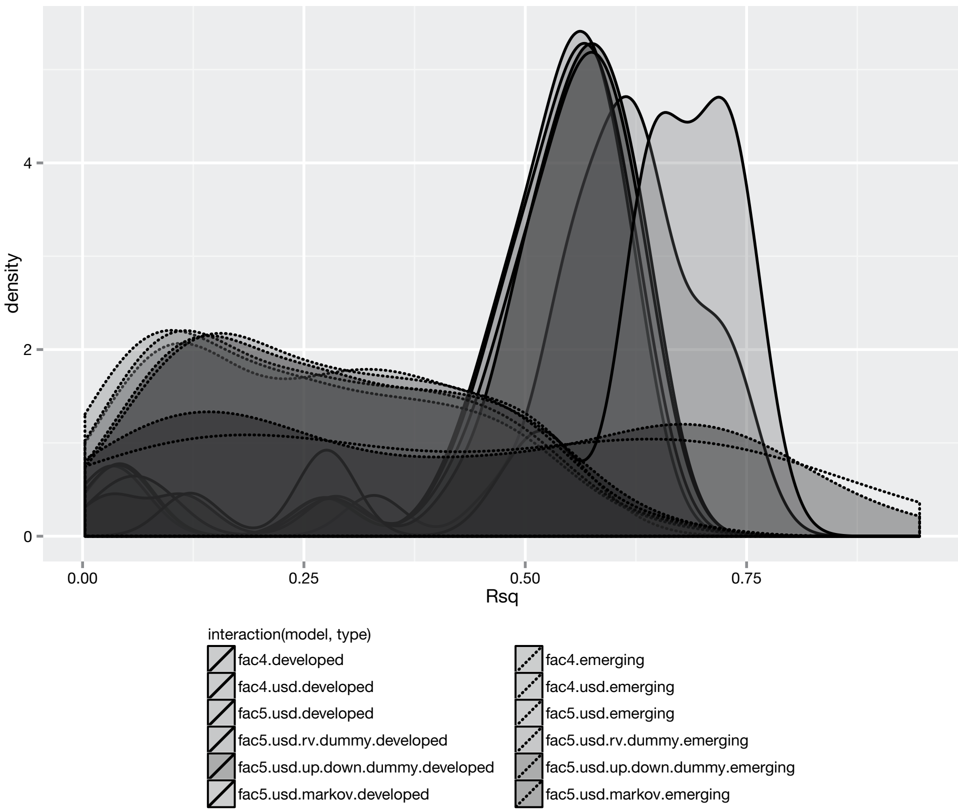

Figure 18. Kernel density estimates of R-squared coefficients for six models from the first part of research, separately for developed and for emerging countries

Note: Density estimates of R-squared coefficients for emerging markets are marked with dotted lines, while density estimates of R-squared for developed countries are marked with solid lines. We can see that density estimates of R-squared for developed countries are centered around much higher values in comparison to density estimates for emerging ones.

Source: authors' own calculations.

To supplement the visual analysis with statistical rigor, we applied tests for differences in means and in variances of R-squared coefficients between emerging and developed countries for each model. The results of these tests, together with the respective values of means and variances, are presented in Table 1.

Table 1. Comparisons of average values and variances of R-squared coefficients in developed and emerging countries across models

model	mean			variance		
	developed	emerging	pvalue	developed	emerging	pvalue
fac4	0.582	0.245	0.000	0.033	0.025	0.376
fac4.usd	0.520	0.223	0.000	0.027	0.026	0.825
fac5.usd	0.642	0.306	0.000	0.027	0.037	0.413
fac5.usd.rv.dummy	0.532	0.260	0.000	0.026	0.023	0.686
fac5.usd.up.down.dummy	0.537	0.286	0.000	0.026	0.031	0.580
fac5.usd.markov	0.603	0.333	0.000	0.018	0.028	0.245

Note: Results for Student-t tests for differences in means and F-test for differences in variances of R-squared coefficients across models. We observe that while differences in means are always significant, differences in variances are never significant at 95% significance level.

Source: authors' own calculations.

Such comparison enables us to formulate our first conclusion that the results of six multi-factor models (including regime switching mechanism as well) do not differ significantly when analysed on per-country basis. This conclusion does not change when we analyse results separately for emerging and for developed countries. However, focusing on the results between the two groups of emerging and developed countries we come to two crucial observations.

First, the highest explanatory power of the five-factor model can be observed for developed equity indices (blue pallete density estimates in comparison to red pallete ones). In this group almost all R-squared values are higher than 50%. On the other hand, for emerging markets they have much lower values. The R-squared coefficients seemingly are characterized by higher dispersion in the case of emerging markets, however, this difference is not statistically significant. This conclusion does not change when we analyse the results of different tested models.

The second issue noticed here, partly connected with the first one, is that multi-factor models are satisfactorily explaining the ERP for developed countries, while they could be misspecified for the emerging markets subgroup. The reason for this difference could be that majority of all models proposed during the last 30 years were prepared for developed countries on the basis of thorough empirical investigations of developed market data, while emerging market data was practically unavailable. In the final part of results section we try to present rationale for this phenomenon.

Taking into account the fact that one of our main outcomes from this part is that results do not differ significantly between tested models, we decided to focus on interpretation of the five-factor model (fac5.usd) which then will be the benchmark model for the second and third part of the research section. Our results for equity indices are in many ways quite similar to well known studies for stock returns (Lieksnis 2010; Davis, Fama, and French 2000), however they do not reveal such strong effects connected with the risk factors as was presented in the literature before.

In order to draw more conclusions with regard to different results for developed and for emerging markets, we analyzed the density estimates of parameters' estimates and R-squared coefficients separately for these two types of equity indices (Figure 19).

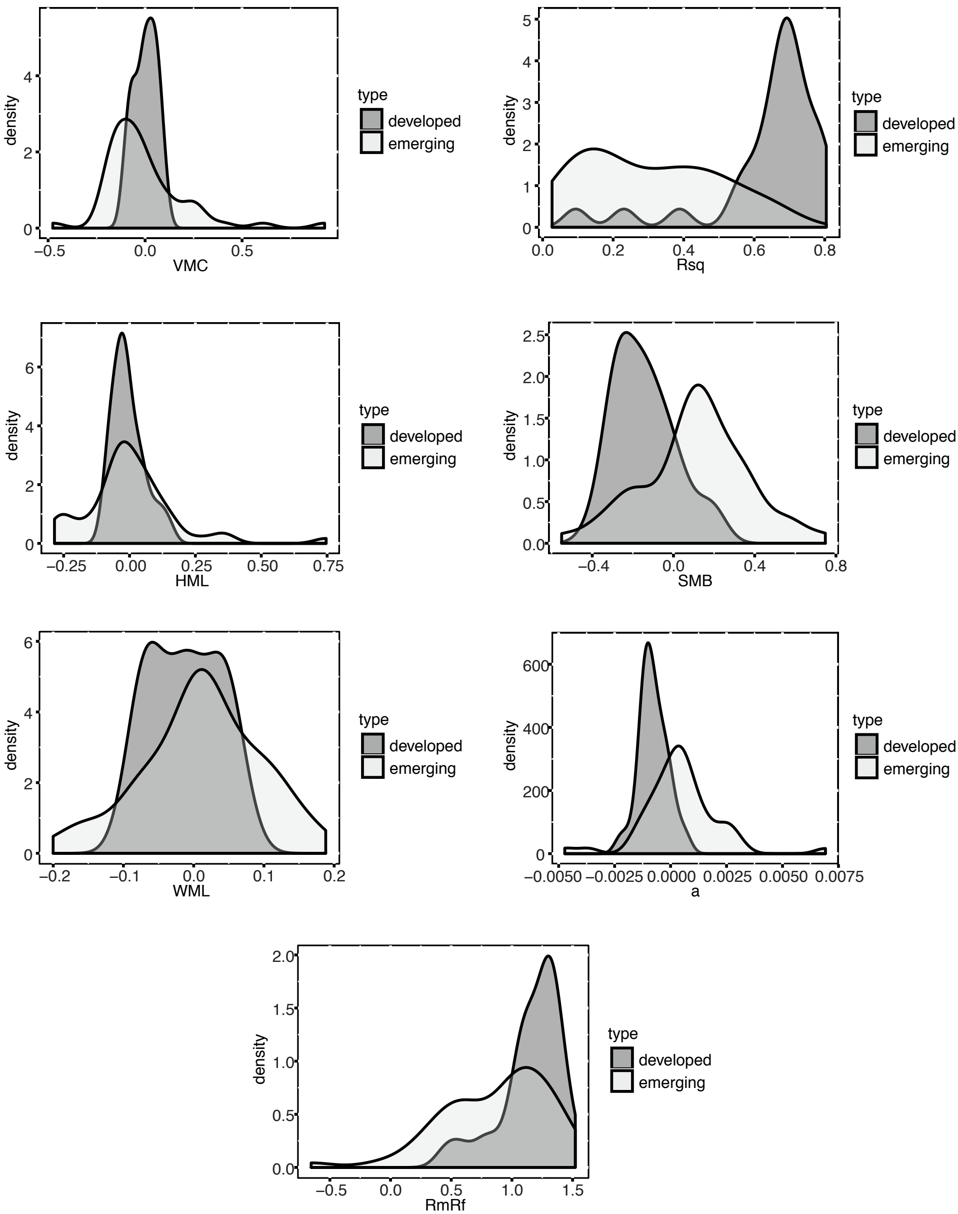

Figure 19. Kernel density estimates of parameters' estimates and R-squared coefficients separately for developed and emerging equity indices

Note: The data covers the period between 2000−2015. Five factor model. Factors based on USD.

Source: authors' own calculations.

Our additional conclusions can be summarized as follows:

1. The results of regressions for developed countries with highest R-squared coefficients have negative (but close to zero) alpha coefficients (significant in 50% of cases) which informs us that there was no excess returns which were not explained by our five-factor model. On the other hand, on average, alpha coefficients for emerging equity indices are positive but still rather insignificant. It could mean that our model for emerging markets requires further investigations.
2. Beta for (R_m−Rf) factor is on average higher for developed countries and additionally less diversified across countries in comparison to emerging markets.
3. The sensitivity to HML factor is similar for the developed and emerging markets, however again, it is much more diversified for the emerging equity indices.
4. The average values of SMB beta are negative for developed countries and lower in comparison to emerging markets, however, their diversity is much higher for emerging markets as before.
5. Characteristics of WML beta estimates are very similar between emerging and developed markets.
6. The only important observation concerning VMC betas is that the values are close to zero and dispersion is much higher among emerging markets just as in the other cases.
7. Separate density estimates for R-squared for developed and for emerging markets confirmed previous observations (based on Figure 18) that regressions for developed markets have significantly higher explanatory power than regressions for emerging markets.

These observations suggest that five-factor model can be a quite robust approach for developed markets and then it is characterized by high explanatory power. However, it should be amended and enhanced with additional risk factors and probably some state variables for emerging markets. We decided to do this in the second part of this research.

4.1.2. Second part of research. Seven-factors models

In the second part of the research we decided to introduce several amendments to our models (new risk factors, common variables and country specific variables described in details in section 2.3). This step resulted in new estimations of our five-factor and seven-factor models. Gaussian kernel density estimates of R-squared coefficients for these new models can be seen on Figure 20.

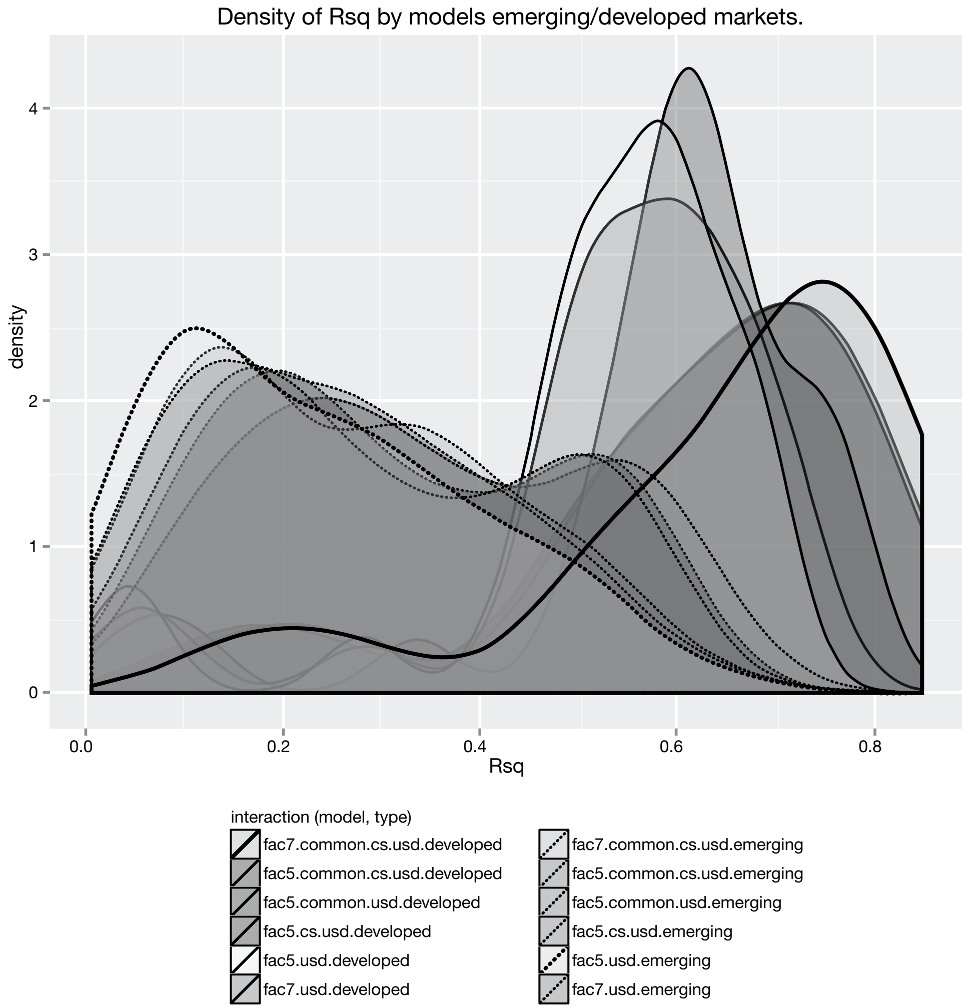

Figure 20. Kernel density estimates of R-squared coefficients for six models from the second part of research, separately for developed and for emerging countries

Note: Kernel density estimates of R-squared coefficients for emerging markets are marked with dotted lines, while density estimates of R-squared for developed countries are marked with solid lines. We can see that density estimates of R-squared for developed countries are centered around much higher values in comparison to density estimates for emerging ones and what is more important density estimates for new models are moved to the right in comparison to the benchmark model from the first part (intentionally indicated with bold dotted line − fac5.usd). The best model is indicated with bold solid line (fac7.common.cs.usd).

Source: authors' own calculations.

Again, just as in the case of initial set of models, we applied tests for differences in means and in variances of R-squared coefficients between emerging and developed countries for each model. Table 2 depicts the summary of the results of these tests together with respective means and variances of the coefficients.

Table 2. Comparisons of average values and variances of R-squared coefficients in developed and emerging countries across models

model	mean			variance		
	deve-loped	**emer-ging**	**pvalue**	**deve-loped**	**emer-ging**	**pvalue**
fac7.common.cs.usd	0.632	0.333	0.000	0.034	0.027	0.417
fac5.common.cs.usd	0.613	0.308	0.000	0.033	0.028	0.579
fac5.common.usd	0.566	0.270	0.000	0.037	0.030	0.521
fac5.cs.usd	0.584	0.265	0.000	0.030	0.025	0.527
fac5.usd	0.642	0.306	0.000	0.027	0.037	0.413
fac7.usd	0.533	0.253	0.000	0.027	0.024	0.689

Note: Results for Student-t tests for differences in means and F-test for differences in variances of R-squared coefficients across models. We observe that while differences in means are always significant, differences in variances are never significant at 95% significance level.

Source: authors' own calculations.

The most important observation based on Figure 20 and Figure 2 is that density estimates for all new models are moved to the right, which means that on average they had higher explanatory power in comparison to the set of models from the first part. The difference in means is statistically confirmed. This feature can be seen both for developed countries and for emerging ones. Moreover, the main conclusions from the first part stating that our models for developed countries have higher explanatory power in comparison to emerging ones is still valid.

We can observe that new set of variables (CV and CspV) had significant influence on our models (density estimates of R-squared for these models are the farthest on the right). We can notice that the best model occurred to be fac7.common.cs.usd i.e. the model with 2 additional risk factors, common variables and country specific variables. What is more important, based on new amendments we can distinguish two separate groups of equity indices among emerging markets. It can be seen in the two-mode shape of R-squared density estimates for emerging countries.

The most convincing explanation of the above phenomenon is that currently we can observe three types of countries when we consider the level of development instead of choosing two subgroups (emerging and developed). The third group consists of countries which according to the most popular classifications are still in emerging markets group (like Poland or South Korea) but they behave rather like developed country and many of their economic indicators and measures are much closer to these for developed ones.

The above observations enticed us to thoroughly analyze density estimates of parameters for our seven factor model with additional variables (fac7.common.cs.usd) what can be seen on Figure 21.

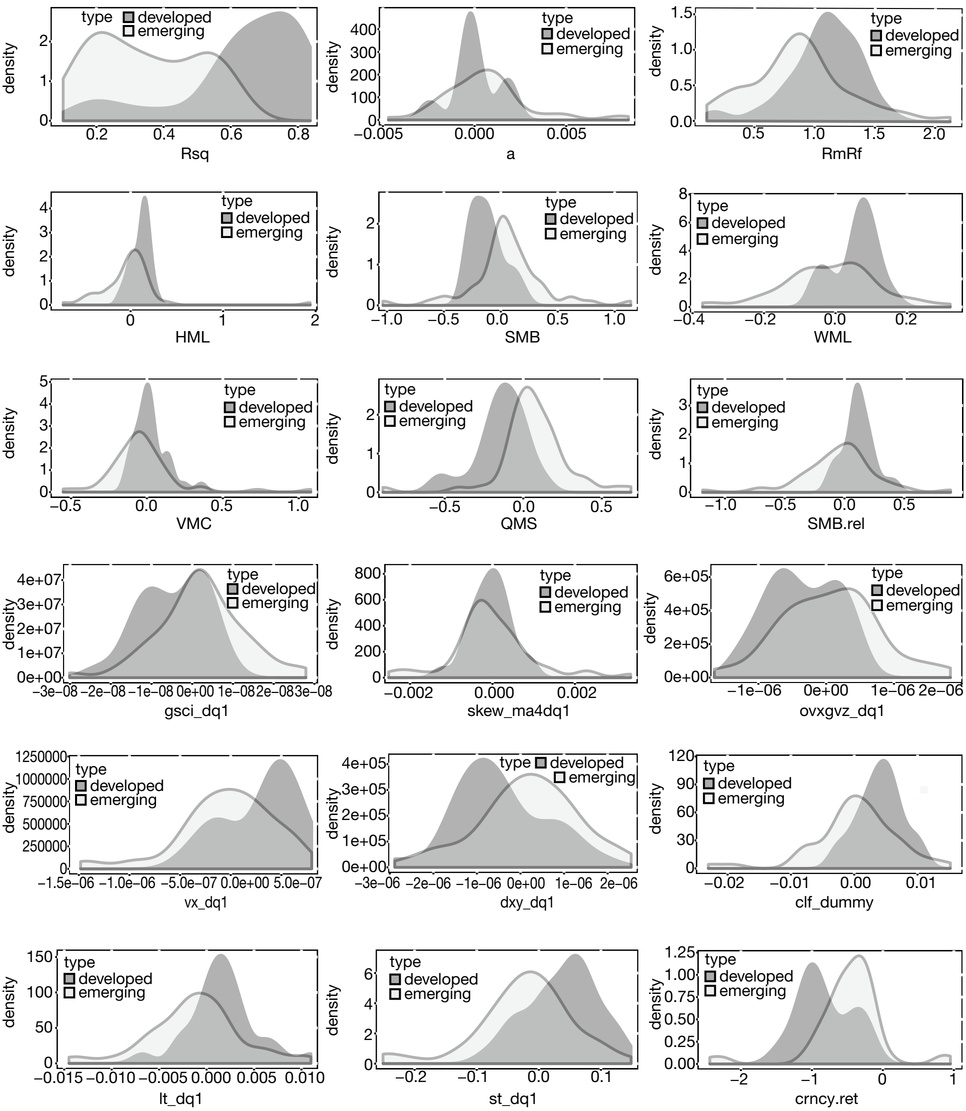

Figure 21. Kernel density estimates of parameter estimates and R-squared for seven-factor model separately for developed and for emerging equity indices

Note: The data cover the period between 2000–2015. Seven-factor model with CV and CspV. Factors based on USD.

Source: authors' own calculations.

The observations based on density estimates of parameters of seven factor model can be compared with these for five-factor model:

1. Kernel density estimates of R-squared for developed and emerging markets confirmed previous observations based on Figure 20 that regressions for developed markets have higher explanatory power than these for emerging markets.

2. Kernel density estimates of alpha parameter did not change significantly.
3. Beta for (R_m-R_f) factor for developed countries is, on average, higher than 1 and higher than average for emerging market countries and higher than beta for (R_m-R_f) for five-factor model.
4. The sensitivity to HML factor for developed markets, is higher than for five-factor model, while we do not observe any significant differences in comparisonto emerging markets.
5. The average values of SMB beta did not change at all.
6. Characteristics of WML beta estimates for developed countries is on average higher than WML beta for emerging ones.
7. The average values of VMC beta did not change at all.
8. The average values of QMS beta did not differ significantly between developed and emerging market countries.
9. Characteristics of SMBrel beta estimates for developed countries is on average higher than SMBrel beta for emerging markets and additionally it is less dispersed.
10. Analyzing the sensitivity to common and country specific variables we can say that there were many similarities between emerging and developed countries.

Summarizing the results from the second step we can say that the new variables and the risk factors add value but main conclusions from the first step still remain valid. In the next section we will analyze results of Markov switching models described in section 2.3.3.

4.1.3. Third part of research. Seven factor Markov switching models

The third part of results is devoted to Markov switching models based on the same set of variables as used in the second part. Figure 22 presents Gaussian kernel density estimates of R-squared coefficients for seven Markov switching models analyzed in this section.

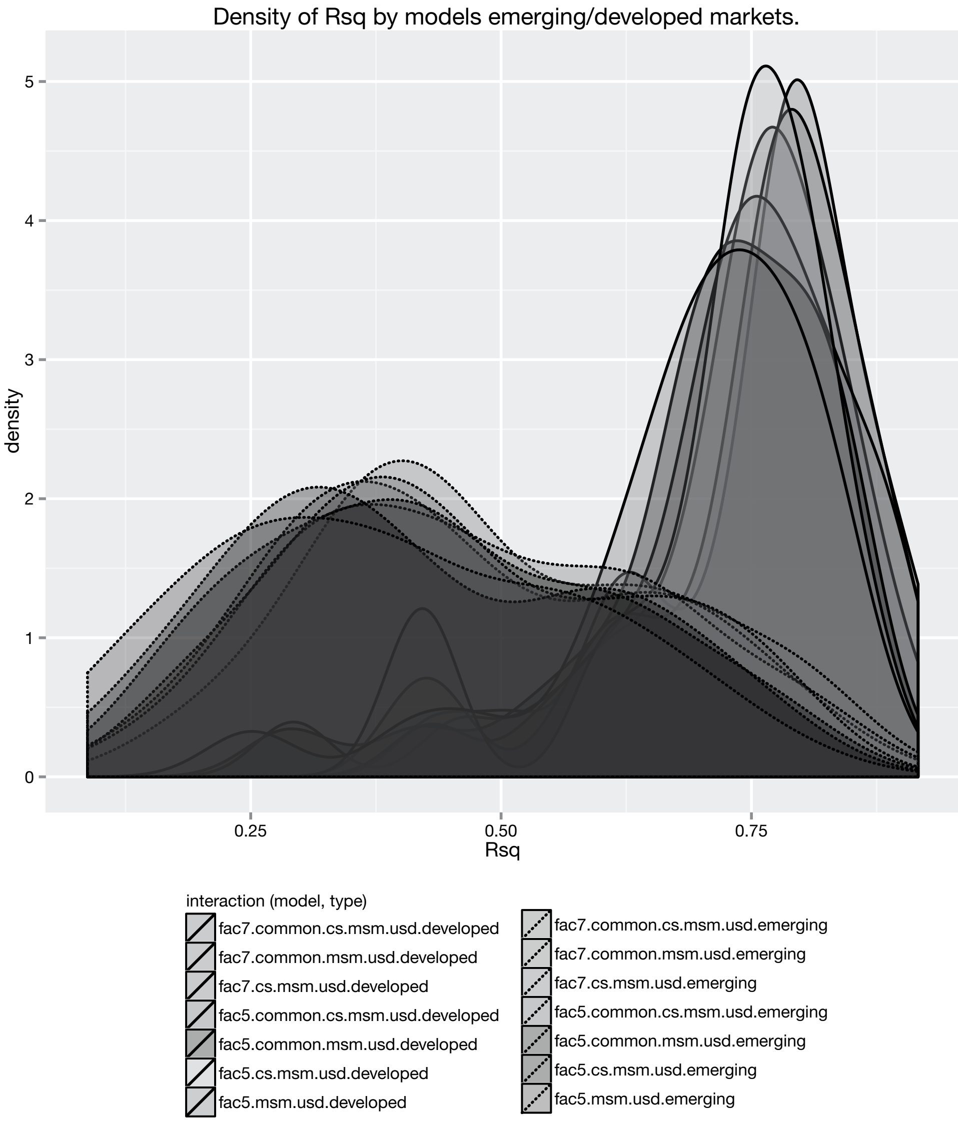

Figure 22. Kernel density estimates of R-squared coefficients for seven Markov switching models, separately for developed and for emerging countries

Note: Density estimates of R-squared coefficients for emerging markets are marked with dotted lines, while kernel density estimates of R-squared for developed countries are marked with solid lines. We can see that density estimates of R-squared for developed countries are centered around much higher values in comparison to density estimates for emerging ones.

Source: authors' own calculations.

The results of statistical tests for differences in means and in dispersion of R-squared coefficients between the two analyzed groups of countries are presented in Table 3.

Table 3. Comparisons of average values and variances of R-squared coefficients in developed and emerging countries across models

model	mean			variance		
	developed	emerging	pvalue	developed	emerging	pvalue
fac7.common.cs.msm.usd	0.770	0.599	0.000	0.017	0.022	0.477
fac7.cs.msm.usd	0.666	0.375	0.000	0.024	0.029	0.646
fac7.common.msm.usd	0.764	0.593	0.000	0.018	0.022	0.530
fac5.commonź	0.693	0.432	0.000	0.025	0.024	0.622
fac5.common.msm.usd	0.732	0.531	0.000	0.019	0.028	0.240
fac5.cs.msm.usd	0.713	0.522	0.000	0.020	0.027	0.293
fac5.msm.usd	0.603	0.333	0.000	0.018	0.028	0.245

Note: Results for Student-t tests for differences in means and F-test for differences in variances of R-squared coefficients across models. We observe that while differences in means are always significant, differences in variances are never significant at 95% significance level.

Source: authors' own calculations.

It may seem that the results for Markov switching models are much less dispersed inside groups of emerging and developed countries. However, just as in all previous cases, the difference in variances is not statistically significant. Again, we can observe much higher average R-squared values for developed countries. The second important issue in the case of the results of the third part is that after adding new variables and risk factors to our model the previous advantage of Markov switching models observed in the first section is not so obvious. We can even say that the difference between R-squared presented in Figure 22 is indeed insignificant.

In the next section we will try to explain why our results are so different in comparison to these observed for multi-factor single stock models for the last several decades.

4.2. Explanation of results

Having analyzed our results we can ask a natural question why they are different than those presented in the literature for single stocks. We would like to investigate why our multi-factor models are not able to fully explain variability of excess returns for the emerging markets.

We see four possible explanations for such results.

The first one is the different time span. In most previous studies the data covered more than 80% of bull markets − from late 1960s until the beginning of 2000s (Figure 23). Contrary to that, in our research we had two distinct bull markets (2003−2007 and 2009−2015) and two distinct bear markets (2000−2003 and 2007−2009) what resulted in rather horizontal long-term trend during the last 15 years. This could be the reason for substantially lower R-squared in our research when compared with the results of studies for equity stock returns.

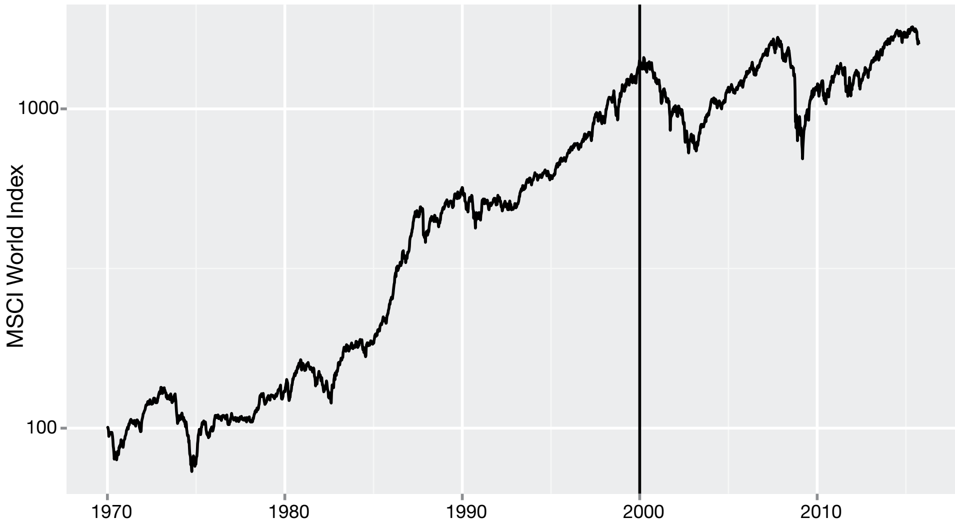

Figure 23. Dynamics of MSCI World Index 1969−2015

Note: This chart presents weekly data for MSCI Index over the period between 1969−2015. MSCI Index is cap weighted index for global equity indices.

Source: authors' own calculations.

The second reason could be different explanatory power of risk factors in a strong upward trend versus up and down movements in the horizontal trend. This is well illustrated on Figure 24, where behavior of zero-cost portfolios built basedon our risk factors is presented with comparison to fluctuations of broad index (Rm.cum).

The third explanation is that most of the previous studies mainly used data for developed markets. Moreover, various modifications of multi-factor models (i.e., additional factors, functional form) were introduced based on the analysis of such data, what in our opinion could illustrate strong over-fitting bias and model risk.

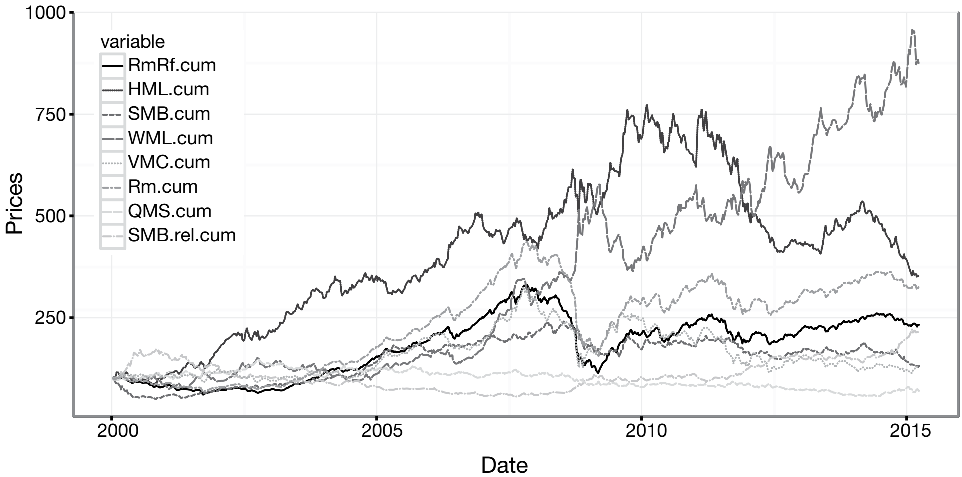

Figure 24. Risk factors from the seven-factor model versus market index returns (R_m)

Note: This chart presents risk factor dynamics over the period 2000−2015 for the seven-factor model.

Source: authors' own calculations.

Lastly, the reason could also be different time frequency. We use weekly data instead of monthly data, as we want to explain variability of excess returns on more frequent basis.

The interpretation presented above is only a possible explanation of different results obtained in our research. However, if it turns out to be the correct one, then it is a very convincing example of data over-fitting problem and model risk which has been repeated for quite a long period of time.

5. Summary

It is important to emphasize that these results are only the first part of rather larger attempt to fully understand reasons behind the variability of world equity indices excess returns with special attention on various behavior of developed markets in comparison to emerging ones.

The most surprising issue concerning our results is that the differences among various multi-factor models are not substantial but the attempt to add new risk factors and additional variables showed that there is some field for model improvements.

Our second conclusion is that, we observe substantial differences between model explanatory power for developed and emerging markets. On one hand, in this group almost all R-squared values are higher than 50%. On the other hand, for emerging markets they get much lower values. This conclusion does not differ when we analyse the results of different tested models.

These two points lead us to our major hypothesis which is at least partially confirmed: multi-factor models contain a satisfactory set of factors in the case of developed countries, while they could be misspecified for the emerging markets subgroup. Therefore, we claim that the results for emerging market equity indices require further investigation and future research should be mainly focused on two issues.

The first one is the continuous search for additional factors. There are numerous ideas present in the single stock literature that are not yet researched in our current context. This includes, for example, liquidity risk (Rahim and Noor 2006; Liu 2004), return-on-equity effects, earning surprises or macro surprises, systemic risk, liquidity risk or betting-against-beta effects (Frazzini and Pedersen 2014)). Of course, the obtained model should be sparse and robust. Secondly, researchers should concentrate on the novel model implementations concerning its functional form and introducing state variables. Our research confirms that both of these paths lead to some improvement in the quality of estimates as well as increase in explanatory power. Two additional factors − percentage deviation from nominal GDP and relative capitalization factor − help better explain variability of risk premia, in particular when combined with additional macroeconomic variables. It seems, however, that the most important effect, especially in the case of developed markets, is brought about by addition of Markov regime switches.

To conclude, we believe that further research should additionally address the questions whether:

- sensitivities to risk factors are stable during various phases of economic cycles;
- correlations among international equity markets differ between high and low volatility periods;
- we can build a zero investment portfolio with positive alpha based on analyzed risk factors.

References

Ammann, Manuel, and Michael Verhofen. 2006. “The Effect of Market Regimes on Style Allocation.” *Financial Markets and Portfolio Management* 20 (3): 309−337.

Angelidis, Timotheos, and Nikolaos Tessaromatis. 2014. Global Portfolio Management Under State Dependent Multiple Risk Premia.” In: *Proceedings of 2nd Economics and Finance Conference, Vienna*, eds. Klara Čermáková and Jiri Rotschedl, 49−82. Prague: International Institute of Social and Economic Sciences and University of Economics, Faculty of Economics, Prague.

Arshanapalli, Bala G., Daniel T. Coggin, and John Doukas. 1998. “Multifactor Asset Pricing Analysis of International Value Investment Strategies.” *Journal of Portfolio Management* 24 (757): 10−23.

Asness, Clifford S. 1995. *The Power of Past Stock Returns to Explain Future Stock Returns*. Unpublished working paper Applied Quantitative Research.

Asness, Cliff, Andrea Frazzini, Aand Lasse H. Pedersen. 2013. Quality minus junk. http://www.econ.yale.edu/~shiller/behfin/2013_04–10/asness-frazzini-pedersen.pdf (accessed: 18.09.2016).

Black, Fischer, Michael C. Jensen, and Myron Scholes. 1972. “The Capital Asset Pricing Model: Some Empirical Tests.” In: *Studies in the Theory of Capital Markets*, ed. Michael C. Jensen. New York: Praeger.

Cakici, Nusret, Frank J. Fabozzi, and Sinan Tan. 2013. “Size, Value, and Momentum in Emerging Market Stock Returns.” *Emerging Markets Review* 16: 46−65.

Carhart, Mark M. 1997. “On Persistence in Mutual Fund Performance.” *Journal of Finance* 52 (1): 57−82.

Chen, Long, Robert Novy-Marx, and Lu Zhang. 2011. “An Alternative Three-Factor Model.” https://papers.ssrn.com/sol3/papers.cfm? abstract_id=1418117&rec=1&srcabs=1567616&alg=1&pos=8 (accessed: 18.09.2016).

Connor, Gregory, and Sanjay Sehgal. 2001. “Tests of the Fama and French Model in India.” *London School of Economics and Political Science Discussion Paper* 379: 1−23.

Davis, James L., Eugene F. Fama, and Kenneth R. French. 2000. “Characteristics, Covariances, and Average Returns: 1929 to 1997.” *Journal of Finance* 55 (1): 389−406.

Donadelli, Michael, and Lorenzo Prosperi. 2011. *The Equity Risk Premium: Empirical Evidence From Emerging Markets*. https://papers.ssrn.com/sol3/papers.cfm? abstract_id=1893378 (accessed: 18.09.2016).

Duarte, Fernando, and Carlo Rosa. 2015. *The Equity Risk Premium: A Review of Models*. https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr714.pdf (accessed: 18.09.2016).

Fama, Eugene F., and Kenneth R. French. 1992. “The Cross-Section of Expected Stock Returns.” *Journal of Finance* 47 (2): 427−465.

Fama, Eugene F., and Kenneth R. French. 2012. “Size, Value, and Momentum in International Stock Returns.” *Journal of Financial Economics* 105 (3): 457−472.
Fama, Eugene F., and Kenneth R. French. 2015. “A Five-Factor Asset Pricing Model.” *Journal of Financial Economics* 116 (1): 1−22.
Foye, James, Dušan Mramor, and Marko Pahor. 2013. “A Respecified Fama French Three-Factor Model for the New European Union Member States.” *Journal of International Financial Management & Accounting* 24 (1): 3−25.
Frazzini, Andrea, and Lasse Heje Pedersen. 2014. “Betting against Beta.” *Journal of Financial Economics* 111 (312417): 1−25.
Griffin, John M. 2002. “Are the Fama and French Factors Global or Country Specific? ” *Review of Financial Studies* 15: 783−803.
Hammerschmid, Regina, and Harald Lohre. 2014. *Regime Shifts and Stock Return Predictability*. https://papers.ssrn.com/sol3/papers.cfm? abstract_id=2445086&rec=1&srcabs=2505732&alg=1&pos=10 ((accessed: 18.09.2016).
Hou, Kewei, Andrew G. Karolyi, Bong Chan Kho. 2011. “What Factors DriveGlobal Stock Returns? ” *Review of Financial Studies* 24: 2527−2574.
Jegadeesh, Narasimhan, and Sheridan Titman. 1993. “Returns to Buying Winners And Selling Losers: Implications for Stock Market Efficiency.” *Journal of Finance* 48 (1): 65−91.
Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny. 1994. “Contrarian Investment, Extrapolation, and Risk.” *Journal of Finance* 49 (5): 1541−1578.
Lieksnis, Raimonds. 2010. “Multifactor Asset Pricing Analysis of the Baltic Stock Market.” *Ekonomika* 89 (4): 85−95.
Lintner, John. 1965. “The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets.” *Review of Economics and Statistics* 47 (1): 13−37.
Liu, Weimin. 2004. *Liquidity Premium and a Two-Factor Model*. EFA 2004 Maastricht Meetings Paper 44, September 2003, 0−52.
Rahim, Ruzita Abdul, and Abu Hassan Shaari Mohd Noor. 2006. “Comparison between Fama and French Model and Liquidity-Based Three-Factor Models in Predicting the Portfolio Returns.” *Asian Academy of Management Journal and Finance* 2 (2): 43−60.
Sakowski, Paweł, Robert Ślepaczuk, and Mateusz Wywiał. 2015. “Cross-sectional Returns from Diverse Portfolio of Equity Indices with Risk Premia Embedded.” *Quantitative Methods in Economics* 16 (2): 89−101.
Sakowski, Paweł, Robert Ślepaczuk, and Mateusz Wywiał. 2016a. “Cross-sectional Returns with Volatility Regimes from Diverse Portfolio of Emerging and Developed Equity Indices.” *eFinanse* 12 (2): 23−35.
Sakowski, Paweł, Robert Ślepaczuk, and Mateusz Wywiał. 2016b. “Do Multifactor Models Produce Robust Results? Econometric and Diagnostic Issues in Equity Risk Premia Study.” *Studia Ekonomiczne. Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach* (forthcoming).

Sharpe, William F. 1964. "Capital Asset Prices: A Theory Of Market Equilibrium Under Conditions of Risk." *Journal of Finance* 19 (3): 425−442.

Tan, Min. 2013. *Regime Switching Behaviour of the UK Equity Risk Premium*. Ph.D. thesis, University of Birmingham.

Wu, Xueping. 2002. "A Conditional Multifactor Analysis of Return Momentum." *Journal of Banking and Finance* 26: 1675−1696.