Least-Squares Monte Carlo Simulation for Time Value of Options and Guarantees Calculation

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Abstract

The article presents an application of least-squares Monte Carlo concept to calculation of Time Value of Options and Guarantees − Market Consistent Embedded Value component. Previously used in American-type options' valuation, this method proved to be a very effective and time-saving tool. The paper summarizes analysis performed on the theoretical Open Pension Fund portfolio (based on Polish market average data).

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1. Introduction

The most significant practical implication of widespread changes in regulation of insurance market is the abandonment of deterministic models calculations and heading towards stochastic methods. The upcoming Solvency II Directive (January 2016), planned IFRS 4 Phase II (2018) and *market-consistent* approach, regarded currently as a market standard, explicitly requires considering multiple scenarios' analysis. Stochastic modelling is used to estimate the probability of outcomes within a forecast in order to predict what conditions might be like in different situations. The upcoming legislation changes portend extended and sophisticated calculations from insurers, especially their actuarial departments. In combination with excessive reporting requirements, stochastic modelling generates a bulk of calculations (typically involving millions of scenarios which make it too time-consuming to run). The use of proxy models has become a popular solution to this problem.

In this article, we present a practical application of a simple, yet powerful method originally used for approximating the values of American options, first introduced by Longstaff and Schwartz (2001) – *the Least squares Monte Carlo* (LSM). We believe that this algorithm is likely to find practical use among actuaries and enable them to speed up assets and liabilities' valuation. To understand the intuition behind this approach, recall that at any moment of exercise, the holder of an American option compares the payoff from immediate exercise with the expected payoff from continuation (i.e. no exercising). The approach, proposed by the authors, assumes that it is possible to estimate conditional expectation (for any exercise time) from the cross-sectional information in the simulation by using a simple regression. The optimal exercise strategy along each path can be estimated by evaluating the conditional expectation function for in-the-money paths and comparing it with the value of immediate exercise. Discounting back and averaging obtained values for all paths results in the *Present Value* of the option.

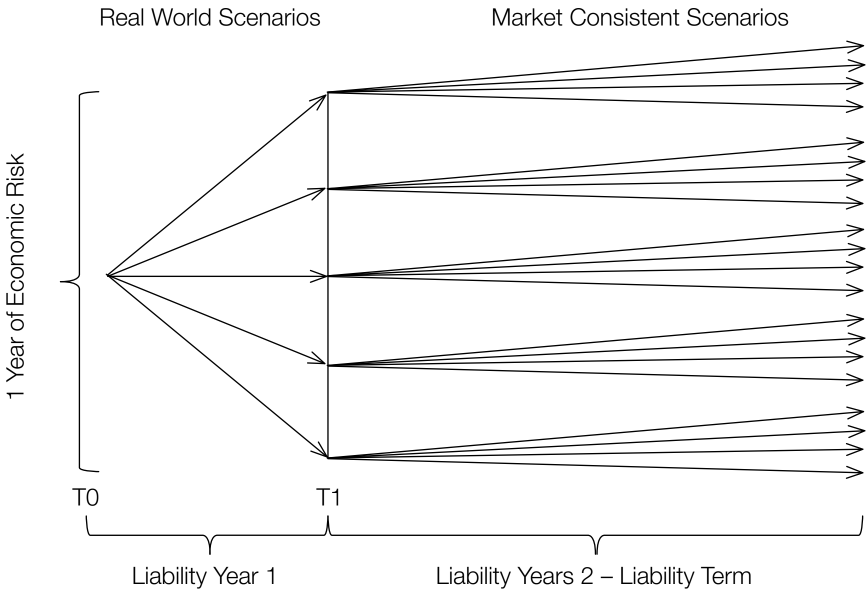
Early attempts of applying *the Least squares Monte Carlo* method to insurance liabilities valuation can be observed in Solvency Capital Requirement (SCR) calculation under Solvency II regime which advises a 1 year 99.5% VaR of net assets measure of capital. Ideally, the value of net assets (assets less liabilities) should be obtained from full probability distribution. In this case, a Monte Carlo simulation is required to value the liabilities. In order to ascertain full distribution in the first year for assets and liabilities, we will encounter a nested stochastic problem. We need to perform a 1 year real world[1] simulation with thousands of scenarios. Within each of these scenarios we need to perform a risk neutral[2] Monte

[1] Based on actual interest rates forecast.

[2] Point of risk neutral pricing is to recover the price of traded options in a way that avoids arbitrage resulting in stocks which only get the risk-free rate.

Carlo simulation with thousands of scenarios to value the assets and liabilities. The required number of scenarios for such a simulation could amount to millions or hundreds of millions. Due to the runtimes of most insurance asset-liability models, this is unfeasible with common hardware. In this case, it is possible to use a proxy model (LSM) which can replace the need to revalue the liabilities with a Monte Carlo simulation in each of the real world scenarios. This approach is easy to implement since nothing more than a simple least squares estimation is required.

Figure 1. Nested stochastic problem



We decided to use *the Least squares Monte Carlo* for *Time Value of Options and Guarantees* (TVOG) calculation – a component of *Market Consistent Embedded Value* (MCEV). CFO Forum[3] advises stochastic TVOG calculation whenever a possible asymmetry in future shareholders' cash flows is identified. In order to forecast TVOG for the year, we will encounter a nested stochastic problem. We need to perform a 1 year real world simulation with thousands of scenarios as in the mentioned SCR valuation. Within each of these scenarios we need to perform a risk neutral stochastic simulation with thousands of scenarios to value the TVOG. Due to extremely long runtime, TVOG forecast is impractical with common hardware, therefore, it

[3] CFO Forum – The European Insurance CFO Forum ('CFO Forum') is a high-level discussion group formed and attended by the Chief Financial Officers of major European listed, and some non-listed, insurance companies. Its aim is to influence the development of financial reporting, value based reporting and related regulatory developments for insurance enterprises on behalf of its members, who represent a significant part of the European insurance industry.

is rarely performed. Not only will *the Least squares Monte Carlo* reduce the duration of calculation but it will save storage space required for thousand scenario files.

Our numerical example will be based on a hypothetical Polish Open Pension Fund created for this analysis and based on market average figures. Calculated figures were benchmarked with available peer group data.

The article is organized as follows: section 2 describes the Polish Pension Fund market context and the theoretical framework of underlying analyses, section 3 presents the methodology of LSM. In part 4 we present a numerical example of the simulation approach, section 5 summarizes the results and contains concluding remarks.

2. Polish Open Pension Funds' market

A pension fund in Poland is a legal entity and its scope of activity includes accumulation and investment of financial resources. Its role is to ensure maximum safety and profitability of investments with the purpose of making benefit payments to fund members after attaining the pensionable age as well as payment of periodic capital pensions. In December 2013, Polish Parliament passed a bill to reform the open pension fund sector in Poland notwithstanding strong criticism from many economists and neglecting warnings that the bill could breach the Polish constitution. The bill entered into force at the beginning of February, and some of its provisions in mid-January 2014. The effects of the bill were observed immediately with the outflow of more than half of the assets held by open pension funds to the Polish state (around PLN 127 bn), a move which some commentators openly call "nationalisation". It is expected that the asset transfer, along with other changes introduced by the bill, will considerably change the Polish capital market.

Asset Management Charge (AMC) levels were not changed by the recently introduced bill and are dependent on the net asset levels as presented in the table below.

Table 1. Asset Management Charges

Net assets (PLN m)		**Monthly fee for management of the open fund on net assets:**
Over	**Up to**	
	8,000	0.045% of Net Assets
8,000	20,000	PLN 3.6 m + 0.040% of Net Assets surplus over 8,000
20,000	35,000	PLN 8.4 m + 0.032% of Net Assets surplus over 20,000
35,000	45,000	PLN 13.2 m + 0.023% of Net Assets surplus over 35,000
45,000		PLN 15.5 m

Source: ACT of 28 August 1997 Law on the Organisation and Operation of Pension Funds.

The construction of Asset Management Charge (AMC) results in asymmetry problems, particularly for stochastic simulations, as in certain scenarios net assets may increase above one of the caps. As a consequence, in order to be in line with MCEV Principles, a TVOG calculation is required.

3. Methodology

In this part we present the theoretical foundations behind our model. Our calculations will be based on the *Least squares Monte Carlo* (LSM) method first introduced by Longstaff and Schwartz (2001) for valuing the American options. The LSM algorithm uses least squares to approximate the conditional expectation function for the value of continuation at different moments in time $t_{K-1}, t_{K-2}, \ldots, t_1$

$$F(\omega; t_k) = E_Q\left[\sum_{j=k+1}^{K} \exp\left(-\int_{t_k}^{t_j} r(\omega, s)ds\right) C\left(\omega, t_j; t_k, T\right) |\mathcal{F}_{t_k}\right] \quad (1)$$

where $C(\omega, t_j; t_k, T)$ is the path of cash flows generated by the option; r (ω, t) is the riskless discount rate, and the expectation is taken conditional on the information set $\mathcal{F}_{tk}$ at time t_k.

The approach is based on the assumption that the unknown functional form of $F(\omega; t_{k-1})$ can be represented as a countable set of measurable basis functions (for proof see Longstaff and Schwartz 2001):

$$F(\omega; t_{k-1}) = \sum_{j=0}^{\infty} a_j L_j(X) \quad (2)$$

Having this setup, one can now approximate $F(\omega; t_{k-1})$ using the finite number (M) of basis function. Finally, we estimate $F_M(\omega; t_{k-1})$ regressing the discounted values of $C(\omega, s; t_{k-1}, T)$ using the basis function paths for which the option is in-the-money.

Stentoft (2004) described thoroughly the theoretical background of the approach and analysed the properties of the estimator giving the mathematical groundwork for the LSM. It was proven that under general conditions the approximation of a set of conditional expectation functions converges to the trueexpectation functions in a multiperiod, multidimensional setting. Moreover, the convergence of price estimates occurs at the true price of the instrument. Quiyi (2009) compared different Monte Carlo methods of pricing American options. The results proved that for multidimentional problems LSM approach is the most suitable choice. The LSM approach gave solid results in the valuation of derivative financial instruments, however, the potential of this method can be also exploited in many different areas.

Sabry and Poulin (2006) as well as Rodrigues and Roche Armada (2006) expanded the use of LSM outside financial options to the real capital investments with many embedded real options and various scenarios. They showed that LSM easily handles complex real world problems within the limited timeframe.

The LSM is also used in the insurance industry for valuing insurance contracts with a surrender option. Andreatta and Corradin (2003) priced the surrender option embedded in the Italian life guaranteed participating policies. The method was further developed by Bernard and Lemieux (2008) who derived the approach to estimate the value of contracts with surrender options including the mortality risk. Bacinello (2003) used the LSM to value the guaranteed life insurance participating policies with periodic premiums and surrender option. Ballotta and Haberman (2002) built a theoretical model for the guaranteed annuity conversion options connected with the unit-linked pension contracts. Despite the fact that such contracts are no longer sold in the UK, the accounting and valuation of such options are challenging for both insurance companies and the regulator. The paper gives a relatively easy and efficient way to estimate the value of the option.

Various authors have already investigated the possibility of using the LSM algorithm for asset valuation under the Solvency II Directive. In Bauer, Bergmann and Reuss (2009), the LSM method is compared to the Nested Simulation Approach, clearly showing superiority of LSM in terms of time required for computation and results' correctness. LSM was more efficient and provided good approximations of the SCR, even though the regression function chosen by the authors is based mostly on their judgment (what is frequently pointed out as the drawback of this method). Koursaris (2011) presents the way in which LSM can be used for Liability Proxy Modelling describing the exact process, model choices, automation and validation of his results. The paper discusses the use of proxy modelling in the context of Solvency II 1 year VaR capital calculation. In another article, Vedani and Devineau (2012) present an alternative examination of LSM in the context of solvency assessment within the Solvency II framework. The authors introduce a possibility of an LSM adaptation on a multi-year time horizon, in order to assess the Overall Solvency Needs. Moreno and Navas (2003) analysed the robustness of the method with respect to the basis function chosen for the regression. According to their work, the approach gives similar results despite different choices of basis functions.

The application of the method in different variations can also be found in e.g. Teguia *et al.* (2014), Dimitrakopoulos (2013), Hörig and Leitschkis (2012). All papers present reasonable outcomes of calculating the solvency requirements taking into consideration both the accuracy of the results and time needed for the computation. Clearly, after adequate amendments, the abovementioned models could be easily applied to the calculation of TVOG, which is the focus of this paper. In the following section, we present our approach to the LSM application for TVOG for Polish Pension Fund system.

In our model, we focus mainly on the value of Assets under Management (AuM) as a sole determinant of AMC and TVOG. This approach lets us limit the model complexity to a minimum and speed up the possible model recalculations. We reduced the number of parameters and as a result, the outflow variables are mortality, lapses and the "slider" mechanism introduced with the new bill. The "slider" assumes that 10 years before reaching the retirement age the assets will be gradually moved to the Social Insurance system lowering the AuM in Polish Pension Fund. The inflows to AuM are premiums (lowered by the premium charge) and investment return on assets. The asset value at the end of period can be expressed as:

$$A_t = [A_{t-1} + P_t(1-p)](1 - AMC_{t-1}) \\ \left(1 - {}_1q_{x+t-1}\right)(1 - s_t)(1 + r_t), 1 \leq t \leq T \tag{3}$$

where,

A_t stands for Assets under Management.
P is in time t, p is the charge on premium
AMC_t as defined in table 1
${}_1q_{x+t-1}$ is the probability of death or lapse of a person aged $x + t-1$ in the next year s_t is the outflow due to the „slider" *machanism as a percentage of the whole portfolio r_t is the investment income*

This simplified model allows us to determine the value of AuM and AMC for all moments t. In the next chapter the results of our calculations will be presented.

4. Numerical example

4.1. Stochastic approach

Contrary to deterministic modelling based on the investment returns, according to their most likely estimate stochastic modelling assumes processing multiple economic scenarios. The final product is a distribution of outcomes, which shows not only the most likely valuation, but also the confidence intervals. The most likely estimate is given by the probability density function centre of mass which is typically also the peak (mode) of the curve (in case of symmetric distributions). In case of asymmetry, stochastic valuations vary from deterministic valuation based on an average scenario and the TVOG is the resulting difference.

The generally accepted market practice is to develop insurance models based on per policy approach (computation is processed for each single policy). It means 1.3 million calculations for an average pension fund portfolio deterministic valuation (1.3 million members). The number of economic scenarios used for stochastic modelling varies from 500 to 10 000 resulting in 650 million to 13 billion model-recalculations. Optimization tools such as policy grouping[4] or flexing[5] are available but reduction in the number of operations means a decrease in valuation accuracy. Projecting future TVOG value requires additional bulk of calculation – respectively 325 billion and 320 trillion calculations (500 and 10000 scenarios) – nearly impossible to process within reasonable time and hardware constraints.

For the purpose of this article we prepared two versions of TVOG model: one developed in Excel (using VBA macros) – the most commonly used software and one in R – free software environment for statistical computing. The reason for building two identical models in different programming languages is to compare model running times and calculation stability – factors essential for such comprehensive modelling. We decided to create a simple model covering only variables required for TVOG calculation that is Assets under Management (AuM) determinants. We assumed a single policy approach – the complete portfolio is grouped to one average record. Thereby, the deterministic calculation lasts less than 1 second. In the table below we present models processing time for selected types of valuation.

Table 2. Comparison of durations for calculated models

Model	Deterministic (single scenario)	Deterministic (1000 scenarios)	Stochastic (10 000 scenarios)
Excel (VBA)	*<1 sec.*	*19 sec.*	*210 sec.*
R	*< 1 sec.*	*22 sec.*	*220 sec.*

Source: own estimation.

In our calculation we used the following assumptions:

1) **Assets under Management** – PLN 34 billion is above the market average (PLN 11 billion) but the asymmetry effect is emphasised (AuM will reach PLN 45 billion cap).
2) **Premium** – PLN 1 billion based on own calculation (adequate for companies with AuM = PLN 34 billion).
3) **Lapse rate** – 0.5% due to prohibition of acquisition introduced in 2012, the lapse effect is almost negligible.

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[4] Grouping is usually based on policy attributes like age, gender, duration etc. (but sometimes also on obtained results). The method is based on multiplying single policy results by the number of policies with coherent attributes. Grouping is very effective for pension funds because of calculation simplicity and the limited number of attributes, however, it leads to asymmetry effect blurring.

[5] Flexing determines the impact of the most likely changes in economic assumptions on each model variable (in deterministic calculation) and applies obtained proportions to calculate other economic scenarios (stochastic).

4) **Wage inflation** – 4.0% based on own estimation.
5) **Mortality tables** – Polish Mortality Tables 2013 with 50% loading (based on market benchmark).
6) **Asset mix** – 70% equity, 30% bond based on current reports.
7) **Assets Management Charge** – in line with legislation (please refer to Table 1).
8) **Premium Charge** – 1,75% based on market average.

For the purpose of generating economic scenarios, we used a simple Vasicek model (1977). The assumptions used for the purpose of calibrating interest rate simulations are based on Zero-Coupon Treasury Bonds (10-years) return:
1) 2013/12/31: 4.356%;
2) 2014/12/31: 2.531%.

We decided to process three types of calculations: TVOG as of 2013/12/31 based on actual assumptions, TVOG as of 2014/12/31 forecastand TVOG as of 2014/12/31 also based on actual assumptions (to verify forecast quality). In the table below we present models processing times for selected types of valuations.

Table 3. Stochastic calculation results (PLN)

Present Value Asset Management Charge (PV AMC)	2013/12/31 (actual)	2014/12/31 (forecast)	2014/12/31 (actual)
Deterministic	*2 486 866 375*	*2 703 357 326*	*2 811 566 667*
Stochastic (1000 scenarios)	*2 265 205 654*	*2 362 749 115*	*2 460 101 013*
TVOG (difference)	***-221 660 721***	***-340 608 211***	***-351 465 654***
TVOG (%)	***-8.91%***	***-10.34%****	***-12.50%***

*Value calculated as an average TVOG among forecast scenarios, hence the average TVOG (%) is not equal to the average TVOG value.
Source: own estimation.

Definitions:
1) **Present Value Asset Management Charge (PV AMC)** – discounted value of Assets Management Charge on the calculation date.
2) **Deterministic** – deterministic valuation based on the most likely estimation (single economic scenario).
3) **Stochastic** – average valuation of 1000 model calculations based on economic scenario set.
4) **TVOG** – measure of asymmetry, difference between deterministic and stochastic valuation.

In case of TVOG forecast, we encounter nested stochastic problem. We needed to perform 1000 valuations based on 1000 economic scenarios each – 1 million model recalculations. In the table below we present models processing times for forecast valuation:

Table 4. Comparison of durations of calculated forecasts

Scenarios	***1000 scenarios***	***2000 scenarios***	***5000 scenarios***	***10000 scenarios***
Operations	***1 million***	***4 million***	***25 million***	***100 million***
Excel (VBA)	*7 h*	*28 h (estimated*)*	*174 h (estimated*)*	*698 h (estimated*)*
R	*7 h*	*28 h (estimated*)*	*174 h (estimated*)*	*698 h (estimated*)*

*Assuming that the program is capable of processing that many operations.
Source: own estimation.

4.2. Least squares Monte Carlo approach

As we have illustrated in the previous paragraph, in order to obtain accurate results, the Nested Simulations Approach requires a large number of simulations; therefore it is very time-consuming. As a consequence, this approach may not be feasible for more complex specifications. For *the Least Squares Monte Carlo* approach, on the other hand, considerably fewer simulations are needed to obtain accurate results. The drawback of this method lies in the choice of the regression function.

For the purpose of this analysis we decided to calibrate three types of regression functions. First of all, we want to benefit from stochastic calculations described in the previous section. Additionally, we processed other TVOG valuations on a hypothetical set of assumptions and regressed the results in order to emphasise the fact that *the Least squares Monte Carlo* is an independent forecasting tool. Following is a precise description of the analysis:

1) regression based on stochastic forecast results – 1000 valuations, each based on 1000 economic scenarios,
2) regression based on 2000 valuations with modified AuM, Premium, economic scenarios (modified base rate and volatility),
3) regression based on 200 valuations randomly selected from the set of 2000 mentioned in the previous point.

The endogenous variable in our regressions is TVOG as a percentage of Present Value Asset Management Charge. The natural choice of exogenous variables are: interest rate, AuM and volatility.

In the table below we present TVOG forecast (2014/12/31) based on three regressions:

Table 5. Calculation results (PLN)

TVOG (%)	Stochastic	Regression 1	diff.	Regression 2	diff.	Regression 3	diff.
2013/12/31 (actual)	*-8.91%*	*-10.56%*	*-19%*	*-9.62%*	*-8%*	*-8.96%*	*-1%*
2014/12/31 (forecast)	*-10.34%*	*-10.34%*	*0%*	*-10.45%*	*-1%*	*-9.62%*	*7%*
2014/12/31 (actual)	*-12.50%*	*-12.43%*	*1%*	*-9.85%*	*21%*	*-9.24%*	*26%*

Source: own estimation.

The Least squares Monte Carlo approach results proved to be a good approximation of the TVOG value. The regression based on the nested stochastic simulations forecast does not vary significantly from the findings based on traditional methods. Independently assessed regression functions (based on 2000 and 200 valuations generated for the purpose of regression fitting) delivered similar results. Therefore we believe that reduction in number of simulations is possible.

5. Conclusion

The purpose of this article is to present a new effective and simplified method of calculation for the Time Value of Options and Guarantees within the framework of Solvency II. We compared two concepts of asymmetry valuation: the generally accepted market practice – stochastic modelling and *the Least squares Monte Carlo* approach. While assessing the TVOG forecast with the traditional, scenario-based technique we encountered a nested stochastic problem and the calculations were time-consuming. The LSM approach is considerably more effective and provides satisfying approximations of TVOG.

The significant impact of the choice of the regression function can be seen as a drawback of this method. Additionally, the decision-making process is required to be repeated whenever the nature of asymmetry may be affected e.g. in case of new legislation. On the other hand, once defined, the regression function can be used in multiple forecasts (even for more distant reporting dates). We believe that LSM method can be reported to financial authorities without being perceived as a *black-box* model which is frequent for actuarial models.

Another promising direction for future research is the combination of both discussed approaches. The regression based on nested stochastic calculation results proved to be accurate. Therefore, we believe that the desired solution could be based on a 1-year stochastic and 2+ years regression forecasts – employing an iterative scheme. Additionally, as we have demonstrated, it is expected to obtain better estimates with small number of simulations.

Finally, in the long run, we believe that advanced numerical approaches as those presented in this paper, should allow a computationally feasible and sufficiently accurate assessment of TVOG not only for Open Pension Funds but wherever asymmetrical distributions can be found.

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